16) $f(x)=2x^3+x^2-13x+6$

$0=(x-2)(2x^2+5x-3)$

$0=(x-2)(2x-1)(x+3)$

$x-2=0 \quad 2x-1=0 \quad x+3=0$

$\boxed{x=2,\ \frac{1}{2},\ -3}$

$\boxed{f(2)=0}$

$$\begin{array}{r|rrrr} 2 & 2 & 1 & -13 & 6 \\ & & 4 & 10 & -6 \\ \hline & 2 & 5 & -3 & 0 \end{array}$$

17)

c) $\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}$

d) $0=(x-2)(2x^2+5x-3)$

$0=(x-2)(2x-1)(x+3)$

$x-2=0 \quad 2x-1=0 \quad x+3=0$

$x=2 \quad x=\frac{1}{2} \quad x=-3$

$f(2)=0 \ \therefore \ x-2=\text{factor}$

$$\begin{array}{r|rrrr} 2 & 2 & 1 & -13 & 6 \\ & & 4 & 10 & -6 \\ \hline & 2 & 5 & -3 & 0 \end{array}$$

e) y-INT $\rightarrow x=0$

$f(0)=6 \quad (0,6)$

f) APPR. CRITICAL PT.

$f(-1)=18 \quad (-1,18)$

$f(1.25)=-4.78$

≈ -5

18) $x=2 \quad x=2-3i \quad x=2+3i$

$f(x)=k(x-2)(x-(2-3i))(x-(2+3i))$

$=k(x-2)(x-2+3i)(x-2-3i)$

$f(x)=k(x-2)(x^2-4x+13)$

$f(1)=-10$

$-10=k(-1)(10)$

$-10=-10k$

$k=1$

$f(x)=(x-2)(x^2-4x+13)$

$\boxed{f(x)=x^3-6x^2+21x-26}$

19) $x=-3,\ x=-3,\ x=i,\ x=-i$

$0=(x+3)(x+3)(x+i)(x-i)$

$0=(x+3)^2(x^2+1)$

$f(-1)=16$

$f(x)=k(x+3)^2(x^2+1)$

$16=k(4)(2)$

$16=8k$

$2=k$

$\boxed{f(x)=2(x+3)^2(x^2+1)}$

$f(x)=2x^4+12x^3+20x^2+12x+18$

Pre-Calculus Practice Test 2.4-2.5

Assg.# Key

Name:______________________ Date:________________ Per:_______

NO GRAPHING CALCULATORS ALLOWED. SHOW ALL THE WORK.

Use long division to find the quotient of:

1) $(x^3-2x^2-5x+6)\div(x-3)$
2) $(x^4-81)\div(x+5)$
3) $(18x^4+9x^3+3x^2)\div(3x^2+1)$

II. Use synthetic division to find the quotient of:

4) $(3x^2+7x-20)\div(x+5)$
5) $(6x^5-2x^3+4x^2-3x+1)\div(x-2)$
6) $(x^7+x^5-10x^3+12)\div(x+2)$

III. Use the Remainder Theorem to find the remainder:

7) $(x^3-7x^2+5x-6)\div(x-3)$
8) $(x^4-5x^3+5x^2+5x-6)\div(x+2)$

IV. Use the Remainder Theorem & Factor Theorem to determine

9) If $(x-3)$ is a factor of $3x^3-2x^2-5x+1$
10) If $(x+2)$ is a factor of $5x^3+10x^2-5x+10$

V. Solve the polynomial functions:

11) $f(x)=2x^3-5x^2+x+2$
12) $f(x)=12x^3+16x^2-5x-3$
13) $f(x)=x^4-6x^2-8x+24$
14) $f(x)=x^4-2x^3+x^2+12x+8$

15) Fill in the table with all the possible combinations for the zeros for the following polynomial function:

$$f(x)=2x^5-3x^3-5x^2+3x-1$$

Possible Positive Real Zeros	Possible Negative Real Zeros	Possible Imaginary Zeros
3	2	0
3	0	2
1	2	2
1	0	4

16) Solve the following polynomial function, given that 2 is a zero
$f(x)=2x^3+x^2-13x+6$

Answers:

1) x^2+x-2
2) $x^3-5x^2+25x-125+\frac{544}{x+5}$
3) $6x^2+3x-1+\frac{-3x+1}{3x^2+1}$
4) $3x-8+\frac{20}{x+5}$
5) $6x^4+12x^3+22x^2+48x+93+\frac{187}{x-2}$
6) $x^6-2x^5+5x^4-10x^3+10x^2-20x+40+\frac{-68}{x+2}$
7) $f(3)=-27$
8) $f(-2)=60$
9) NO
10) NO
11) $x=-\frac{1}{2},1,2$
12) $x=-\frac{3}{2},-\frac{1}{3},\frac{1}{2}$
13) $x=2$ (mult 2), $-2\pm i\sqrt{2}$
14) $x=-1$ (mult 2), $2\pm 2i$
16) $x=+\frac{1}{2},2,-3$
17) a) 3 b) 2
c) 2 or 0 d) 1
e) $\pm1,\pm2,\pm3,\pm6,\pm\frac{1}{2},\pm\frac{3}{2}$ f) $(x-2)(2x-1)(x+3)$
g) $x=-3,\frac{1}{2},2$ h) $(0,6)$
18) $f(x)=x^3-6x^2+21x-26$
19) $f(x)=2x^4+12x^3+20x^2+12x+18$

$f(x) = 2x^3 + x^2 - 13x + 6$

a) According to the Fundamental Theorem of Algebra, determine the number of complex zeros.
b) At most how many turns in the graph?
c) According to Descartes' Rule of Signs, determine the number of possible positive real zeros.
d) According to Descartes' Rule of Signs, determine the number of possible negative real zeros.
e) According to the Rational Zero Theorem, list all the possible rational zeros.
f) Factor completely.
g) Determine all the zeros.
h) Determine the y-intercept.
i) Sketch the graph. Graph all the zeros, y-intercept (if any), and critical points (approximate as needed).

x-axes by 1 y-axes by 2

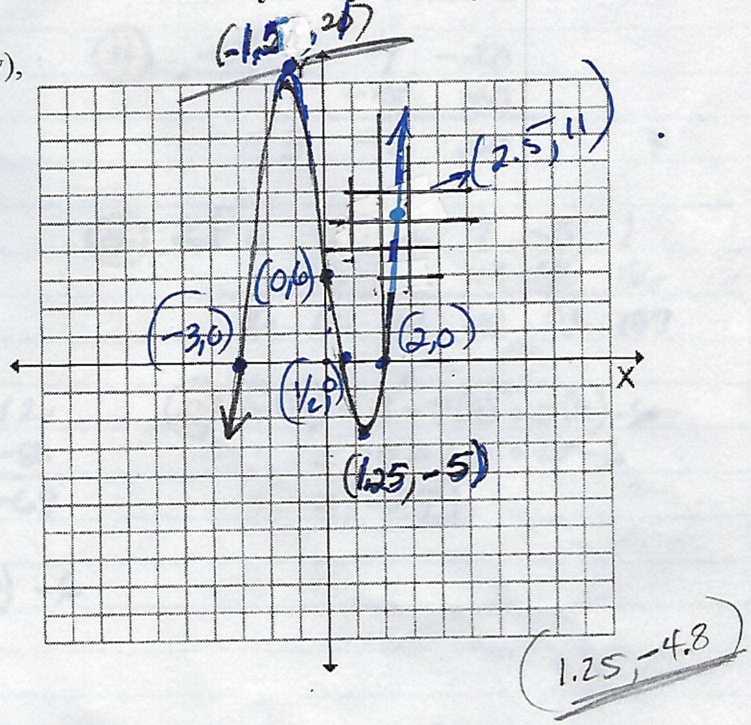
APPRX. CRITICAL PTS: -1, 18

(,) (1.25, -5) min. (-1.5, 21) max

Zeros: x = -3, 1/2, 2

y-INT → (0,6)

x	y
-2	20
-1	18
-1.5	21
1.5	-4.5
2.5	11



18) Find a 3[rd] degree polynomial function f(x) with real coefficients that has 2, and 2-3i as zeros, such that f(1) = –10. Write the answer as a function in descending order.

19) Find a 4[th] degree polynomial function f(x) with real coefficients, i is a zero and -3 is a zero of multiplicity 2, such that f(-1) = 16. Write the answer as a function in descending order.

$$\begin{array}{r} x^2+x-2 \\ x-3\,\overline{)\,x^3-2x^2-5x+6} \\ -\underline{x^3+3x^2} \\ x^2-5x \\ \underline{-x^2+3x} \\ -2x+6 \\ \underline{+2x-6} \\ 0 \end{array}$$

(2)

$$\begin{array}{r} x^3-5x^2+25x-125 \\ x+5\,\overline{)\,x^4+0x^3+0x^2+0x-81} \\ \underline{-x^4-5x^3} \\ -5x^3+0x^2 \\ \underline{5x^2+25x^2} \\ 25x^2+0x \\ \underline{-25x^2-125x} \\ -125x-81 \\ \underline{125x+625} \\ 544 \end{array}$$

(3)

$$\begin{array}{r} 6x^2+3x-1 \\ 3x^2+1\,\overline{)\,18x^4+9x^3+3x^2+0x+0} \\ \underline{-18x^4 \qquad -6x^2} \\ 9x^3-3x^2 \\ \underline{-9x^3 \qquad -3x} \\ -3x^2-3x+0 \\ \underline{+3x^2 \qquad +1} \\ -3x+1 \end{array}$$

(4)

-5	3	7	-20
		-15	40
	3	-8	20

(5)

2	6	0	-2	4	-3	1
		12	24	44	96	186
	6	12	22	48	93	187

(6)

-2	1	0	1	0	-10	0	0	12
		-2	4	-10	20	-20	40	-80
	1	-2	5	-10	10	-20	40	-68

(7) $f(3)=3^3-7(3)^2+5(3)-6$

$=27-63+15-6$

$=\boxed{-27}$

(8) $f(-2)=(-2)^4-5(-2)^3+5(-2)^2+5(-2)-6$

$=16+40+20-10-6$

$=\boxed{60}$

9) $f(3)=3(3)^3-2(3)^2-5(3)+1$

$=81-18-15+1$

$=\boxed{49}$ NO

10) $f(-2)=5(-2)^3+10(-2)^2-5(-2)+10$

$=-40+40+10+10$

$=\boxed{20}$ NO.

11) $f(x)=2x^3-5x^2+x+2$

$0=2x^3-5x^2+x+2$

$0=(x-1)(2x^2-3x-2)$

$0=(x-1)(2x+1)(x-2)$

$x-1=0 \quad 2x+1=0 \quad x-2=0$

$\boxed{x=1 \quad x=-1/2 \quad x=2}$

POSSIBLE RATIONAL ~~FACTORS~~ Zeros:

$\pm\frac{1}{2}, \pm1, \pm2$

$f(1)=2-5+1+2=0$

1 is a zero $\therefore$ $x-1$ is a FACTOR

1	2	-5	1	2
		2	-3	-2
	2	-3	-2	0

12) $f(x) = 12x^3 + 16x^2 - 5x - 3$

$0 = 12x^3 + 16x^2 - 5x - 3$

$0 = (x - \frac{1}{2})(12x^2 + 22x + 6)$

$0 = (x - \frac{1}{2})(3x + 1)(2x + 3)$

$x - \frac{1}{2} = 0 \quad 3x + 1 = 0 \quad 2x + 3 = 0$

$\boxed{x = \frac{1}{2}, -\frac{1}{3}, -\frac{3}{2}}$

<u>Possible Rational Zeros</u>

$\pm 1, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{4}, \pm\frac{1}{6}, \pm\frac{1}{12}$

$\pm 3, \pm\frac{3}{2}, \pm\frac{3}{4}$

$f(\frac{1}{2}) = 0 \quad \therefore x + \frac{1}{2}$ is a factor

$$\begin{array}{r|rrrr} \frac{1}{2} & 12 & 16 & -5 & -3 \\ & & 6 & 11 & 3 \\ \hline & 12 & 22 & 6 & 0 \end{array}$$

13) $f(x) = x^4 - 6x^2 - 8x + 24$

$0 = (x - 2)(x^3 + 2x^2 - 2x - 12)$

$0 = (x - 2)(x - 2)(x^2 + 4x + 6)$

PRIME

$x - 2 = 0 \quad x - 2 = 0 \quad x^2 + 4x + 6 = 0$

$x = 2 \quad x = 2 \quad x = -2 \pm i\sqrt{2}$

$\boxed{x = 2, -2 \pm i\sqrt{2}}$

<u>Possible Rational Zeros</u>

$\pm 24, \pm 12, \pm 6, \pm 8, \pm 3, \pm 4, \pm 2, \pm 1$

$f(2) = 0 \quad \therefore x - 2$ is a factor

$$\begin{array}{r|rrrrr} 2 & 1 & 0 & -6 & -8 & 24 \\ & & 2 & 4 & -4 & -24 \\ \hline & 1 & 2 & -2 & -12 & 0 \end{array}$$

$x^3 + 2x^2 - 2x - 12$

$f(2) = 8 + 8 - 4 - 12 = 0 \quad \therefore x - 2$ is a factor

$$\begin{array}{r|rrrr} 2 & 1 & 2 & -2 & -12 \\ & & 2 & 8 & 12 \\ \hline & 1 & 4 & 6 & 0 \end{array}$$

14) $f(x) = x^4 - 2x^3 + x^2 + 12x + 8$

$0 = (x + 1)(x^3 - 3x^2 + 4x + 8)$

$0 = (x + 1)(x + 1)(x^2 - 4x + 8)$

PRIME

$\underbrace{x + 1 = 0 \quad x + 1 = 0}_{x = -1} \quad x^2 - 4x + 8 = 0$

$x = 2 + 2i$

$\boxed{x = -1, 2 \pm 2i}$

<u>Possible Rational Zeros</u>

$\pm 8, \pm 4, \pm 2, \pm 1$

$f(-1) = 0 \quad \therefore x + 1$ is a factor

$$\begin{array}{r|rrrrr} -1 & 1 & -2 & 1 & 12 & 8 \\ & & -1 & 3 & -4 & -8 \\ \hline & 1 & -3 & 4 & 8 & 0 \end{array}$$

$x^3 - 3x^2 + 4x + 8$

$f(-1) = 0 \quad \therefore x + 1$ is a factor

$$\begin{array}{r|rrrr} -1 & 1 & -3 & 4 & 8 \\ & & -1 & 4 & -8 \\ \hline & 1 & -4 & 8 & 0 \end{array}$$

15) <u>Possible Positive Real Zeros</u>

$+ \; - \; - \; + \; -1 \qquad$ 3 or 1

<u>Possible Negative Real Zeros</u>

$- \; + \; - \; - \; - \qquad$ 2 or 0