# Pre-Calculus
# Notes on 2.6 – RATIONAL FUNCTIONS AND THEIR GRAPHS

***A rational function is basically a division of two polynomial functions.*** That is, it is a polynomial divided by another polynomial. In formal notation, a rational function would be symbolized like this:

$$f(x) = \frac{s(x)}{t(x)}$$

Where s(x) and t(x) are polynomial functions, and t(x) cannot equal zero.

Example: $f(x) = \dfrac{x^2 + x - 20}{x^2 - 3x - 18}$

**<u>Steps for graphing rational functions:</u>**

1. Find the y-intercept, if any, by setting x=0. f(0) = ?

2. Find the x-intercepts by setting y = 0.
   **<u>Note:</u>** if the numerator of any fraction=0, then the fraction =0, thus to find the x-intercepts set the numerator=0 and solve.

3. Find vertical asymptote(s) **(VA)** by setting the denominator =0 and solve. Remember that the answer(s) are equations of vertical lines (x=?)

4. Find horizontal asymptote **(HA)** using the following rules:
   - If the degree of the numerator **<** the degree of the denominator, **then** $\boldsymbol{y = 0}$.
   - If the degree of the numerator **=** the degree of the denominator, **then** $\boldsymbol{y = \frac{LC\ Num.}{LC\ Den.}}$
   - If the degree of the numerator **>** the degree of the denominator, **then NO HA.**

5. Find slant asymptote **(SA):**
   - When does it occur?
     - ✓ If the degree of the numerator **>** the degree of the denominator **<u>BY EXACTLY 1.</u>**
   - How do you find it?
     - ✓ Divide the numerator by the denominator using long division or synthetic division.
     - ✓ Write the answer as y = quotient. Disregard the remainder.
     - ✓ **<u>Note:</u>** Remember linear equations in slope intercept form are written as y=mx + b. Your answer for any slant asymptote will be of that form.

6. Use a table of values to find points between all the vertical asymptotes. Include values of x close to the right and to the left of the VA´s.

7. Graph using the information from steps 1-6.
   **<u>Note:</u>** The asymptotes will make the graph discontinuous, however, it will be smooth with no sharp edges.