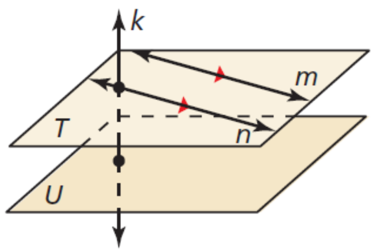
3.1 & 3.2 Notes – Pairs of Lines and Angles

KEY IDEAS

Parallel Lines, Skew Lines, and Parallel Planes

Two lines that do not intersect are either *parallel lines* or *skew lines*. Two lines are **parallel lines** when they do not intersect and are coplanar. Two lines are **skew lines** when they do not intersect and are not coplanar. Two planes that do not intersect are **parallel planes**.



Lines m and n are parallel lines ($m \parallel n$).

Lines m and k are skew lines.

Planes T and U are parallel planes ($T \parallel U$).

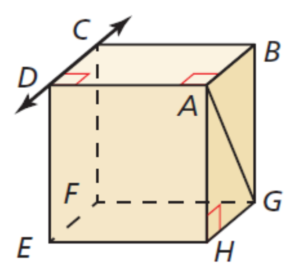
Lines k and n are intersecting lines, and there is a plane (not shown) containing them.

Small directed arrows, as shown in red on lines m and n above, are used to show that lines are parallel. The symbol $\parallel$ means "is parallel to," as in $m \parallel n$.

Segments and rays are parallel when they lie in parallel lines. A line is parallel to a plane when the line is in a plane parallel to the given plane. In the diagram above, line n is parallel to plane U.

Example 1:

Consider the lines that contain the segments in the figure and the planes that contain the faces of the figure. Which line(s) or plane(s) appear to fit each description?



a. line(s) parallel to $\overleftrightarrow{CD}$ and containing point A

b. line(s) skew to $\overleftrightarrow{CD}$ and containing point A

c. line(s) perpendicular to $\overleftrightarrow{CD}$ and containing point A

d. plane(s) parallel to plane EFG and containing point A

SOLUTION

a. $\overleftrightarrow{AB}$, $\overleftrightarrow{HG}$, and $\overleftrightarrow{EF}$ all appear parallel to $\overleftrightarrow{CD}$, but only $\overleftrightarrow{AB}$ contains point A.

b. Both $\overleftrightarrow{AG}$ and $\overleftrightarrow{AH}$ appear skew to $\overleftrightarrow{CD}$ and contain point A.

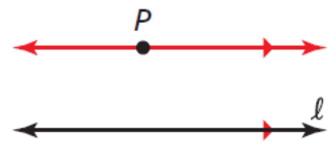
c. $\overleftrightarrow{BC}$, $\overleftrightarrow{AD}$, $\overleftrightarrow{DE}$, and $\overleftrightarrow{FC}$ all appear perpendicular to $\overleftrightarrow{CD}$, but only $\overleftrightarrow{AD}$ contains point A.

d. Plane ABC appears parallel to plane EFG and contains point A.

POSTULATES

3.1 Parallel Postulate

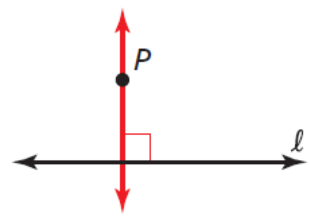
If there is a line and a point not on the line, then there is exactly one line through the point parallel to the given line.



There is exactly one line through P parallel to ℓ.

3.2 Perpendicular Postulate

If there is a line and a point not on the line, then there is exactly one line through the point perpendicular to the given line.



There is exactly one line through P perpendicular to ℓ.

EXAMPLE 2 Identifying Parallel and Perpendicular Lines

The map shows how the roads in a town are related to one another.

a. Name a pair of parallel lines.

b. Name a pair of perpendicular lines.

c. Is $\overleftrightarrow{FE} \parallel \overleftrightarrow{AC}$? Explain.

SOLUTION

a. $\overleftrightarrow{MD} \parallel \overleftrightarrow{FE}$

b. $\overleftrightarrow{MD} \perp \overleftrightarrow{BF}$

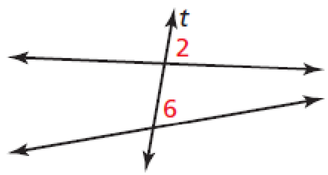
c. $\overleftrightarrow{FE}$ is not parallel to $\overleftrightarrow{AC}$, because $\overleftrightarrow{MD}$ is parallel to $\overleftrightarrow{FE}$, and by the Parallel Postulate, there is exactly one line parallel to $\overleftrightarrow{FE}$ through M.

Identifying Pairs of Angles

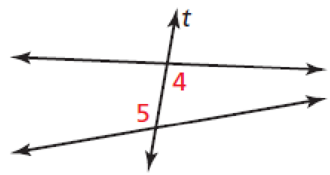
A **transversal** is a line that intersects two or more coplanar lines at different points.

KEY IDEA

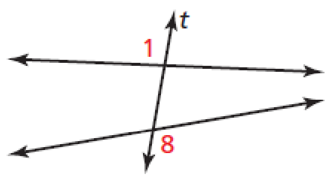
Angles Formed by Transversals



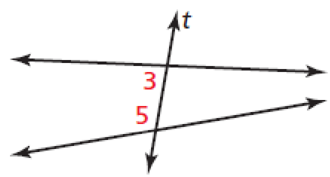
Two angles are **corresponding angles** when they have corresponding positions. For example, ∠2 and ∠6 are above the lines and to the right of the transversal *t*.



Two angles are **alternate interior angles** when they lie between the two lines and on opposite sides of the transversal *t*.



Two angles are **alternate exterior angles** when they lie outside the two lines and on opposite sides of the transversal *t*.



Two angles are **consecutive interior angles** when they lie between the two lines and on the same side of the transversal *t*.

(***Also called Same Side Interior Angles)***

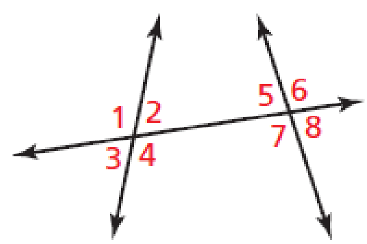
EXAMPLE 3 Identifying Pairs of Angles

WATCH

Identify all pairs of angles of the given type.

a. corresponding **b.** alternate interior

c. alternate exterior **d.** consecutive interior



SOLUTION

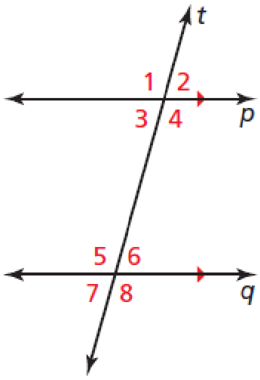
a. ∠1 and ∠5
∠2 and ∠6
∠3 and ∠7
∠4 and ∠8

b. ∠2 and ∠7
∠4 and ∠5

c. ∠1 and ∠8
∠3 and ∠6

d. ∠2 and ∠5
∠4 and ∠7

3.2 - Using Properties of Parallel Lines



THEOREMS

3.1 Corresponding Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.

Examples In the diagram at the left, $\angle 1 \cong \angle 5$, $\angle 2 \cong \angle 6$, $\angle 3 \cong \angle 7$, and $\angle 4 \cong \angle 8$.

Prove this Theorem Exercise 35, page 174

3.2 Alternate Interior Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

Examples In the diagram at the left, $\angle 3 \cong \angle 6$ and $\angle 4 \cong \angle 5$.

Prove this Theorem Exercise 17, page 131

3.3 Alternate Exterior Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent.

Examples In the diagram at the left, $\angle 1 \cong \angle 8$ and $\angle 2 \cong \angle 7$.

Proof Example 4, page 130

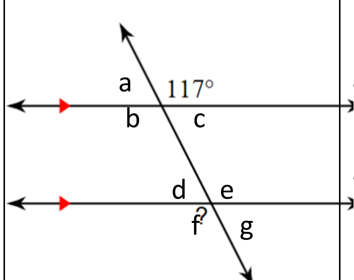
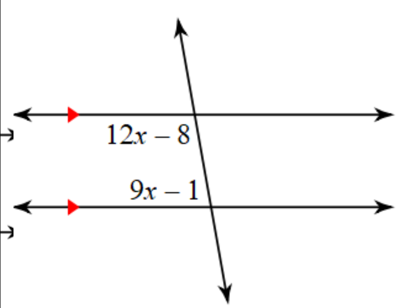
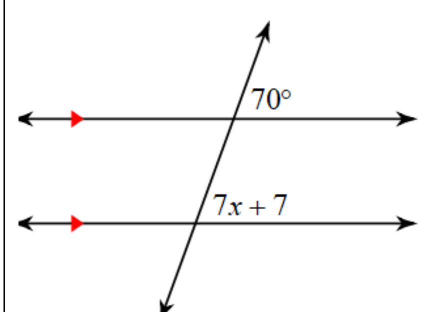
3.4 Consecutive Interior Angles Theorem ***(Also called Same Side Interior Angles)***

If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary.

Examples In the diagram at the left, $\angle 3$ and $\angle 5$ are supplementary, and $\angle 4$ and $\angle 6$ are supplementary.

Prove this Theorem Exercise 18, page 131

Examples:

1) Find the missing angle measures.	2) Find x.	3) Find x.
		

Labels in 1): a, 117°, b, c, d, e, f, g

Labels in 2): $12x - 8$, $9x - 1$

Labels in 3): 70°, $7x + 7$