4.1 Notes – Transformations introduction & Translations

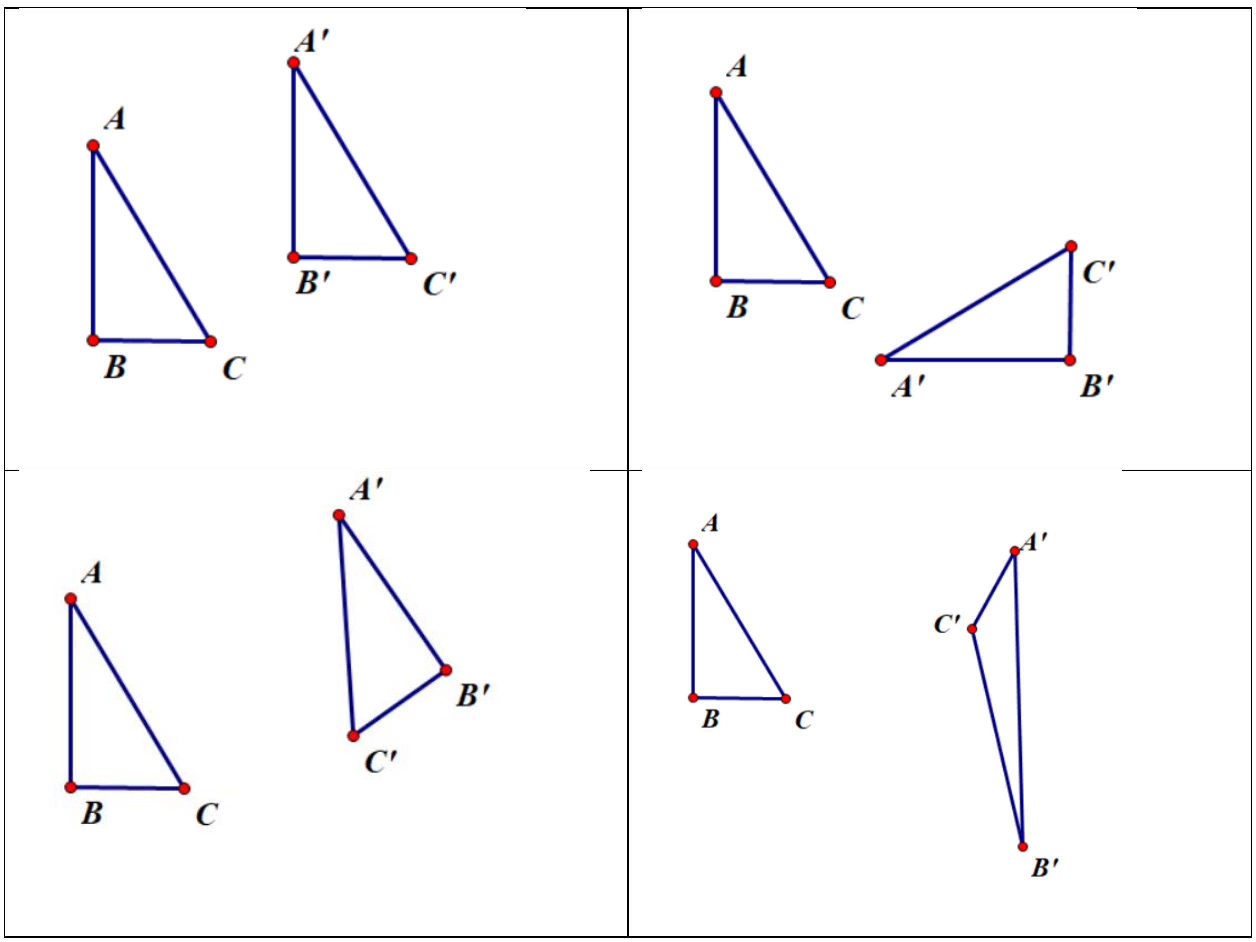
A <u>**transformation**</u> is a function that moves or changes a figure in some way to produce a new figure called an **image**. The original figure is called the **preimage**. The points on the preimage are the inputs for the transformation and the points on the image are the outputs.

We says the preimage is transformed (or **mapped**) to the image. All points in the preimage are indicated with a capital letter, as always, while their corresponding image points are indicated with prime notation. For example, A is mapped to A' (read as *A prime*).

A **rigid motion** is a transformation that preserves size and shape. Another name for a rigid motion is an **isometry**. Rigid motions preserve angle measures, segment lengths, parallelism, among other things to maintain size and shape. A rigid motion maps lines to lines, rays to rays, and segments to segments.

We will be studying three types of rigid motions (isometries) –
TRANSLATIONS (slides), REFLECTIONS (flips) and ROTATIONS (turns).

Ex. Identify the rigid motion performed, if any.

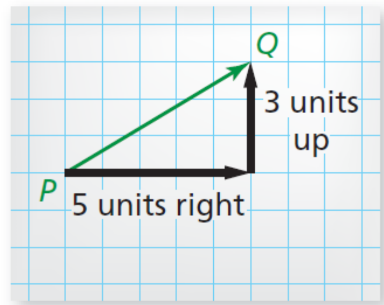


TRANSLATIONS

KEY IDEA

Vectors

The diagram shows a vector. The **initial point**, or starting point, of the vector is P, and the **terminal point**, or ending point, is Q. The vector is named $\overrightarrow{PQ}$, which is read as "vector PQ." The **horizontal component** of $\overrightarrow{PQ}$ is 5, and the **vertical component** is 3. The **component form** of a vector combines the horizontal and vertical components. So, the component form of $\overrightarrow{PQ}$ is $\langle 5, 3 \rangle$.

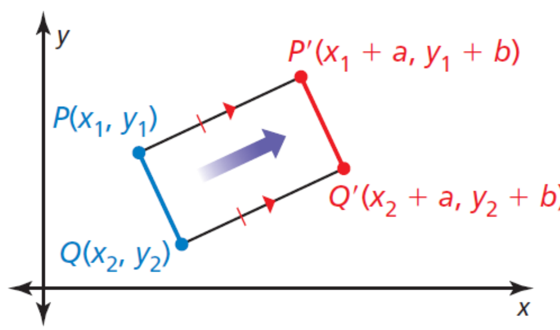


KEY IDEA

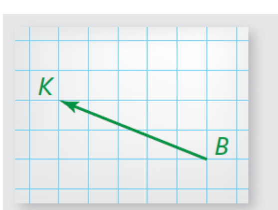
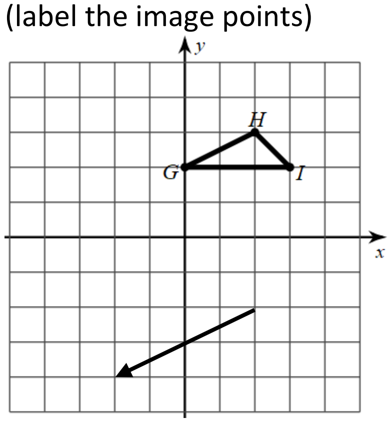
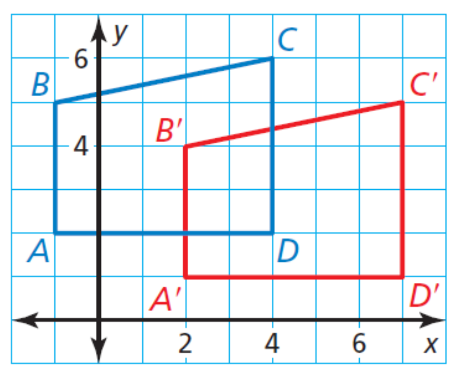
Translations

A **translation** moves every point of a figure the same distance in the same direction. More specifically, a translation *maps*, or moves, the points P and Q of a plane figure along a vector $\langle a, b \rangle$ to the points P' and Q', so that one of the following statements is true.

- $PP' = QQ'$ and $\overline{PP'} \parallel \overline{QQ'}$ or



Examples:

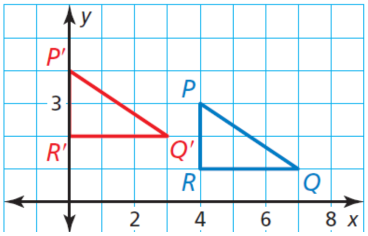
1) Name the vector and write it In component form	2) Translate the figure shown using the given vector then write it in component form (label the image points)	3) State the translation rule in component form AND as draw the vector on the coordinate plane.
 Name:___________ Component form: _______________	 Component form: ____________________	 Component form:__________________

Translation rule format:

You can also express a translation along the vector $\langle a, b \rangle$ using a rule, which has the notation $(x, y) \rightarrow (x + a, y + b)$.

EXAMPLE 3 Writing a Translation Rule

WATCH



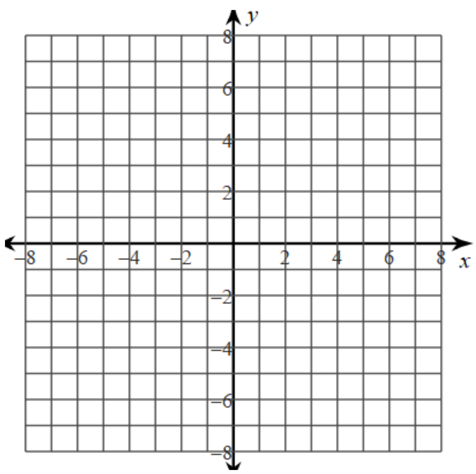
Write a rule for the translation of $\triangle PQR$ to $\triangle P'Q'R'$.

SOLUTION

To go from P to P', move 4 units left and 1 unit up, along the vector $\langle -4, 1 \rangle$.

▶ So, a rule for the translation is $(x, y) \rightarrow (x - 4, y + 1)$.

Ex. Plot the points then perform the indicated translation: 1 unit left and 5 units up
B(-3, -3), C(-1, -1) and D(-1, -4)
Then identify B', C' and D' and state the vector in the specified formats.



B (-3, -3)	C (-1, -1)	D (-1 -4)
B' (,)	C' (,)	D' (,)

Now draw the vector on the graph.

Write the vector in component form:_____________

Write the translation rule:

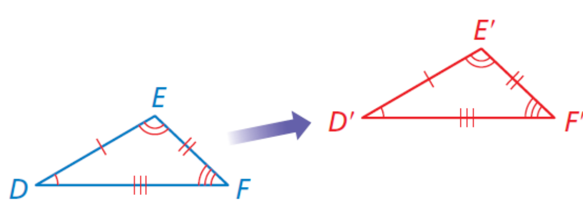
____

POSTULATE

4.1 Translation Postulate

A translation is a rigid motion.

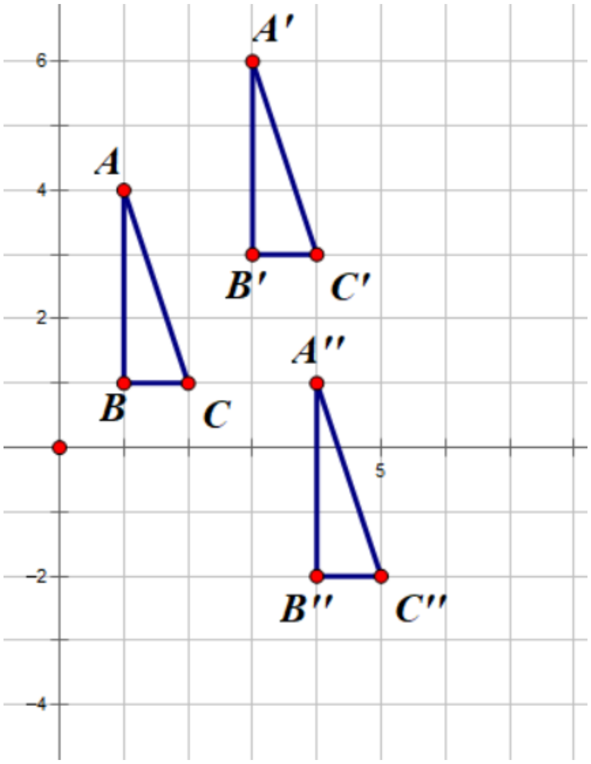
Because a translation is a rigid motion, and a rigid motion preserves length and angle measure, the following statements are true for the translation shown.



- $DE = D'E'$ $EF = E'F'$ $FD = F'D'$
- $m\angle D = m\angle D'$ $m\angle E = m\angle E'$ $m\angle F = m\angle F'$

When two or more transformations are combined to form a single transformation, the result is a **composition of transformations**.

Ex. Note the preimage is triangle ABC, the first image is triangle A'B'C' and the second image is triangle A"B"C".



State the two transformations performed in rule form:

Translation #1:_______________________________

Translation #2:_______________________________

Can these two translations be expressed as one translation? If so, write the rule:

Translation:_________________________________

THEOREM

4.1 Composition Theorem

The composition of two (or more) rigid motions is a rigid motion.

Proof Exercise 34, page 174