4.2 – Reflections

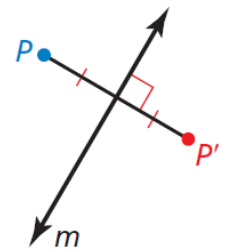
KEY IDEA

Reflections

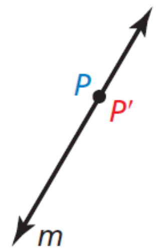
A **reflection** is a transformation that uses a line like a mirror to reflect a figure. This line is called the **line of reflection**.

A reflection in a line m maps every point P in the plane to a point P', so that for each point, one of the following properties is true.

- If P is not on m, then m is the perpendicular bisector of $\overline{PP'}$, or
- If P is on m, then $P = P'$.



point *P* not on *m*



point *P* on *m*

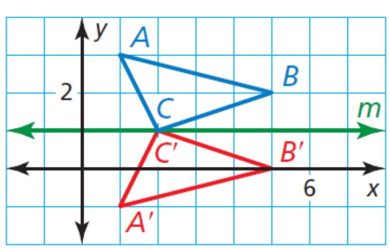
Ex.

Graph triangle ABC with vertices A(1,3), B(5,2) and C(2,1) and its image after the reflection described.

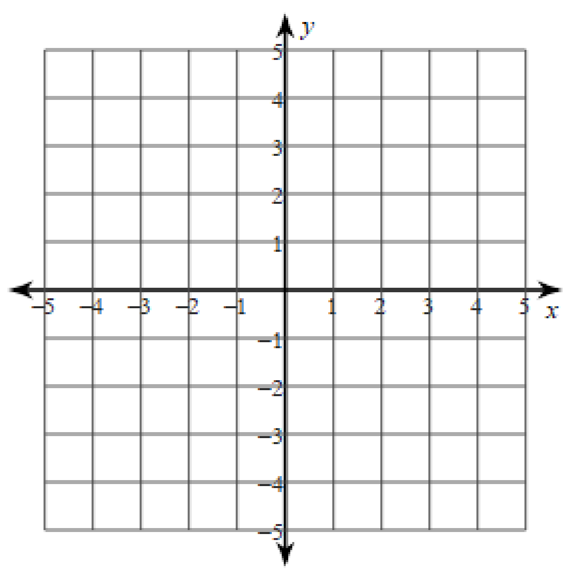
a. In the line m: $y = 1$

SOLUTION

a. Point A is 2 units above line m, so A' is 2 units below line m at $(1, -1)$. Also, B' is 1 unit below line m at $(5, 0)$. Because point C is on line m, you know that $C = C'$.



b. In the line x = 1
(plot the triangle, make the line of reflection and Plot the image of the triangle)

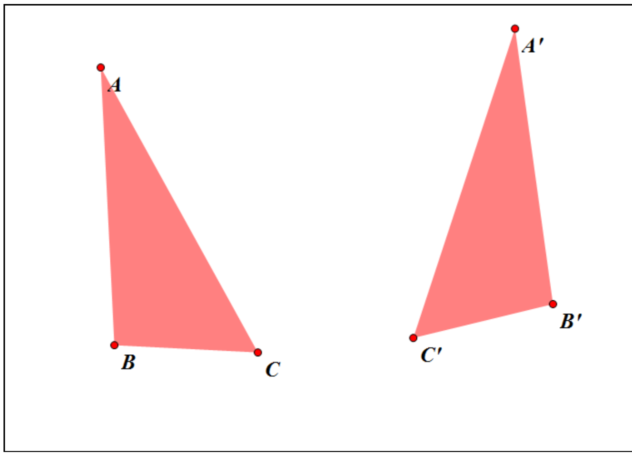


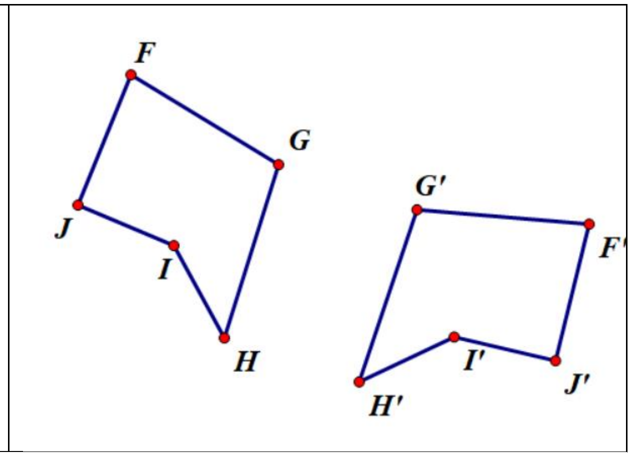
Does the image cross over the line of reflection?

Note the line of reflection is the perpendicular bisector of the segment connecting corresponding points that were not on the line of reflection. In the examples above, the line of reflection is the perpendicular bisector of $\overline{AA'}$ and $\overline{BB'}$.

We can use this to construct the perpendicular bisector given a preimage and its image.

Examples: Construct the line of reflection in each scenario below:

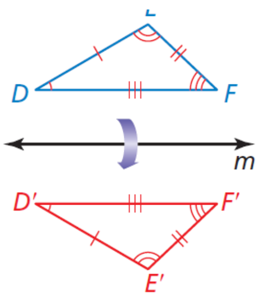




POSTULATE

4.2 Reflection Postulate

A reflection is a rigid motion.



Because a reflection is a rigid motion, and a rigid motion preserves length and angle measure, the following statements are true for the reflection shown.

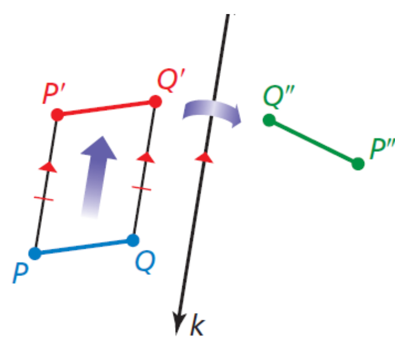
- $DE = D'E'$ $\quad EF = E'F'$ $\quad FD = F'D'$
- $m\angle D = m\angle D'$ $\quad m\angle E = m\angle E'$ $\quad m\angle F = m\angle F'$

Because a reflection is a rigid motion, the Composition Theorem guarantees that any composition of reflections and translations is a rigid motion.

A **glide reflection** is a transformation involving a translation followed by a reflection in which every point P is mapped to a point P'' by the following steps.

Step 1 First, a translation maps P to P'.

Step 2 Then a reflection in a line k parallel to the direction of the translation maps P' to P''.



4.2 Notes continued… Symmetry

Identifying Lines of Symmetry

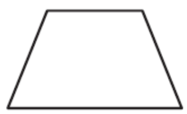
A figure in the plane has **line symmetry** when the figure can be mapped onto itself by a reflection in a line. This line of reflection is a **line of symmetry**, such as line *m* at the left. A figure can have more than one line of symmetry.

EXAMPLE 5 Identifying Line Symmetry

WATCH

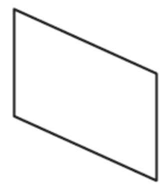
Determine whether each polygon has line symmetry. If so, draw the line(s) of symmetry and describe any reflections that map the polygon onto itself.

a. trapezoid



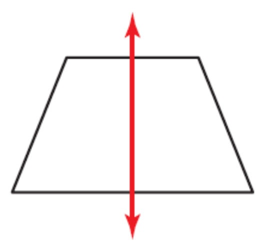
b. regular hexagon

c. parallelogram

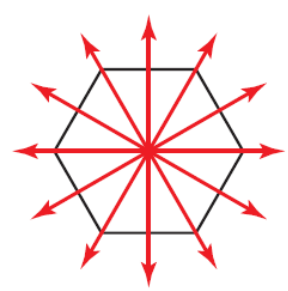


SOLUTION

a. The trapezoid has line symmetry. The line of symmetry is shown. A reflection in the line of symmetry maps the trapezoid onto itself.



b. The regular hexagon has line symmetry. The 6 lines of symmetry are shown. A reflection in any of the lines of symmetry maps the hexagon onto itself.



c. The parallelogram does not have line symmetry because the figure cannot be mapped onto itself by a reflection in a line.

Challenge – Find a company logo with line symmetry