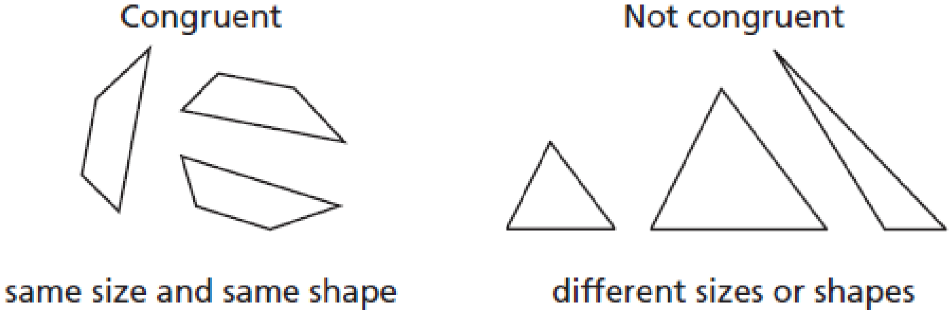
4.4 Congruence and Transformations

Identifying Congruent Figures

Two geometric figures are **congruent figures** if and only if there is a rigid motion or a composition of rigid motions that maps one of the figures to the other. Congruent figures have the same size and same shape.

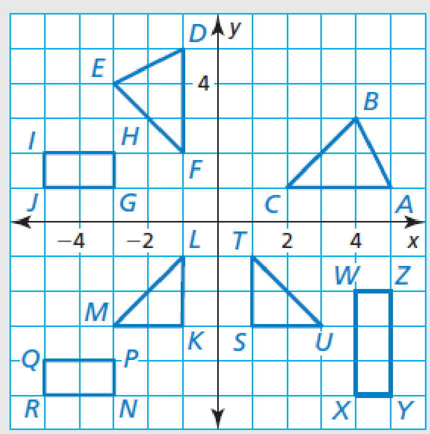


You can identify congruent figures in the coordinate plane by identifying the rigid motion or composition of rigid motions that maps one of the figures to the other. Recall from Postulates 4.1–4.3 and Theorem 4.1 that translations, reflections, rotations, and compositions of these transformations are rigid motions.

Example:

NOTE: THESE SHAPES ARE NOT TRANSFORMED WITH PRIME NOTATION. YOU MUST IDENTIFY CORRESPONDING PARTS CORRECTLY IN YOUR CONGRUENCE STATEMENT!

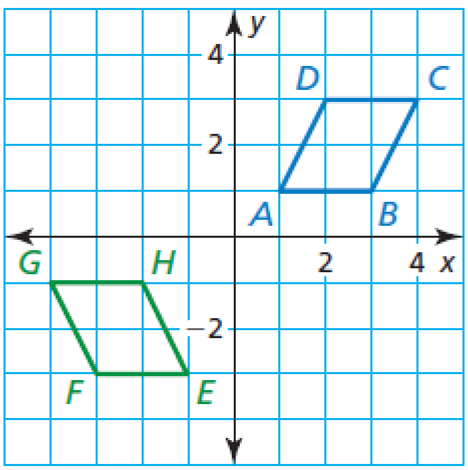
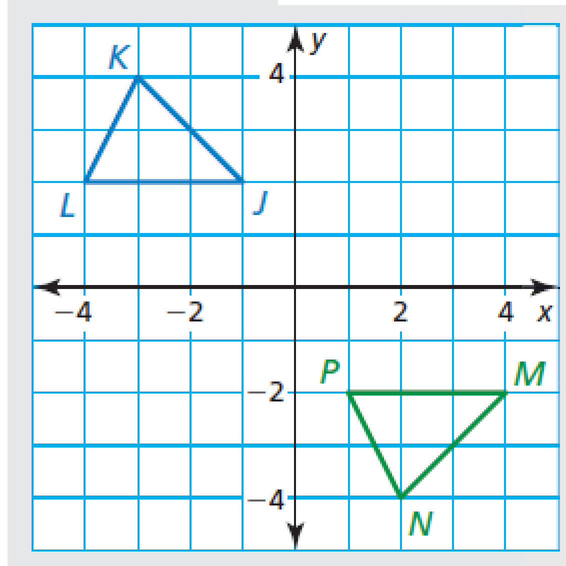
1. Identify any congruent figures in the coordinate plane. Explain.



Congruence Transformations

Another name for a rigid motion or a combination of rigid motions is a **congruence transformation** because the preimage and image are congruent. The terms *rigid motion* and *congruence transformation* are interchangeable.

EXAMPLE 2 Describing a Congruence Transformation

Describe a congruence transformation that maps ▱$ABCD$ to ▱$EFGH$.	Describe a congruence transformation that maps △JKL to △MNP.
	

Using Theorems about Congruence Transformations

Compositions of two reflections result in either a translation or a rotation. A composition of two reflections in parallel lines results in a translation, as described in the following theorem.

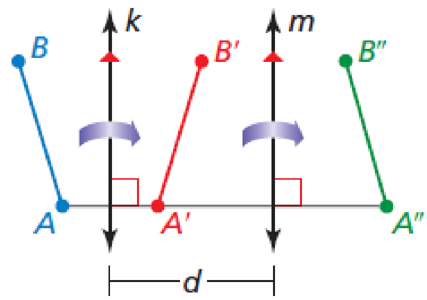
THEOREM

4.2 Reflections in Parallel Lines Theorem

If lines k and m are parallel, then a reflection in line k followed by a reflection in line m is the same as a translation.

If A'' is the image of A, then

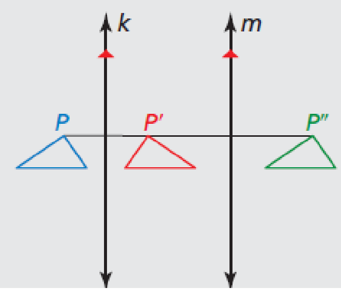
1. $\overline{AA''}$ is perpendicular to k and m, and
2. $AA'' = 2d$, where d is the distance between k and m.



Example:

Use the figure. The distance between line *k* and line *m* is 1.6 centimeters.

6. The preimage is reflected in line k, then in line m. Describe a single transformation that maps the blue figure to the green figure.
7. What is the relationship between $\overline{PP'}$ and line k? Explain.
8. What is the distance between P and P''?



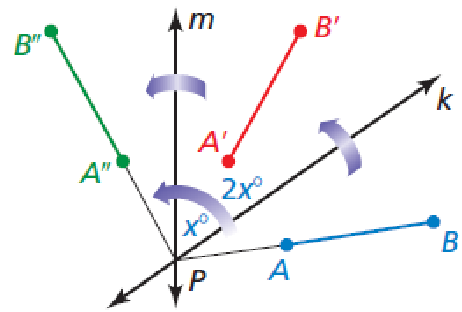
THEOREM

4.3 Reflections in Intersecting Lines Theorem

If lines k and m intersect at point P, then a reflection in line k followed by a reflection in line m is the same as a rotation about point P.

The angle of rotation is $2x°$, where $x°$ is the measure of the acute or right angle formed by lines k and m.

Proof Exercise 22, page 248



$m\angle BPB'' = 2x°$

Example:

9. In the diagram, the preimage is reflected in line k, then in line m. Describe a single transformation that maps the blue figure to the green figure.