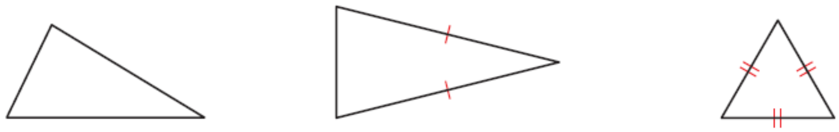
5.1 Angles of Triangles Notes

Remember that a triangle is a three-sided polygon. It can be classified according to its sides & angles.

KEY IDEAS

Classifying Triangles by Sides

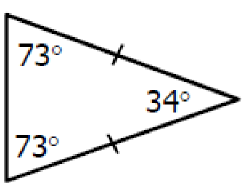
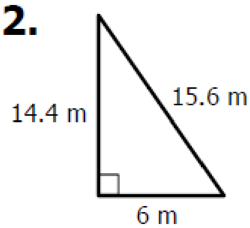
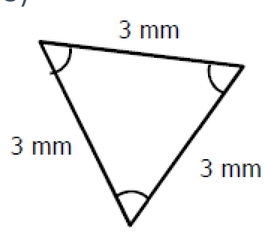
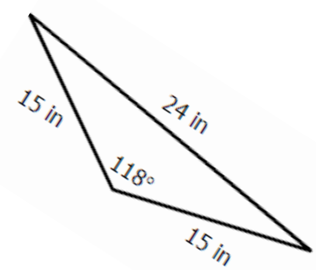
Scalene Triangle	Isosceles Triangle	Equilateral Triangle
no congruent sides	at least 2 congruent sides	3 congruent sides



Classifying Triangles by Angles

Acute Triangle	Right Triangle	Obtuse Triangle	Equiangular Triangle
3 acute angles	1 right angle	1 obtuse angle	3 congruent angles

Examples: Classify each triangle according to its sides and angles.

1)	2.	3)	4)
			
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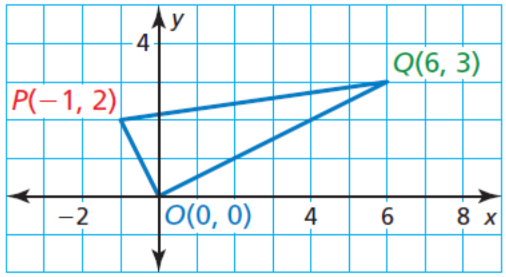
EXAMPLE 2 Classifying a Triangle in the Coordinate Plane

WATCH

Classify $\triangle OPQ$ by its sides. Then determine whether it is a right triangle.



SOLUTION

Step 1 Use the Distance Formula to find the side lengths.

$$OP = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-1 - 0)^2 + (2 - 0)^2} = \sqrt{5} \approx 2.2$$

$$OQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(6 - 0)^2 + (3 - 0)^2} = \sqrt{45} \approx 6.7$$

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{[6 - (-1)]^2 + (3 - 2)^2} = \sqrt{50} \approx 7.1$$

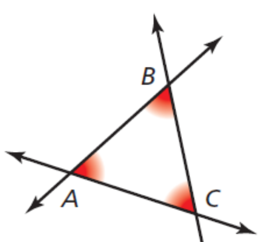
Because no sides are congruent, $\triangle OPQ$ is a scalene triangle.

Step 2 Check for right angles. The slope of $\overline{OP}$ is $\frac{2 - 0}{-1 - 0} = -2$. The slope of $\overline{OQ}$ is $\frac{3 - 0}{6 - 0} = \frac{1}{2}$. The product of the slopes is $-2\left(\frac{1}{2}\right) = -1$. So, $\overline{OP} \perp \overline{OQ}$ and $\angle POQ$ is a right angle.

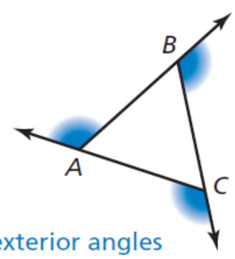
▶ So, $\triangle OPQ$ is a right scalene triangle.

Finding Angle Measures of Triangles

When the sides of a polygon are extended, other angles are formed. The original angles are the **interior angles**. The angles that form linear pairs with the interior angles are the **exterior angles**.



interior angles

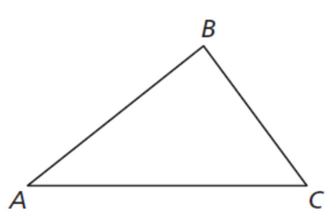


exterior angles

THEOREM

5.1 Triangle Sum Theorem

The sum of the measures of the interior angles of a triangle is 180°.



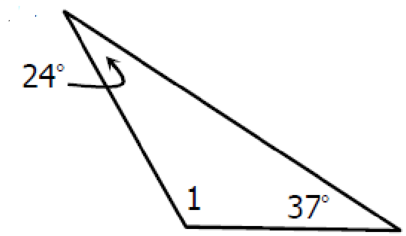
$m\angle A + m\angle B + m\angle C = 180^\circ$

Proof page 232

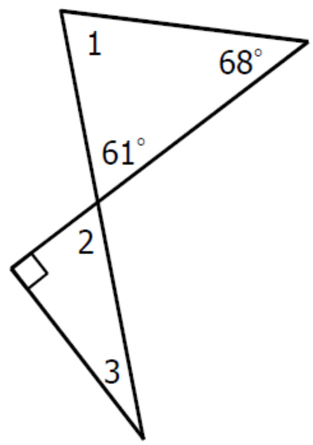
Prove this Theorem Exercise 50, page 236

Examples:

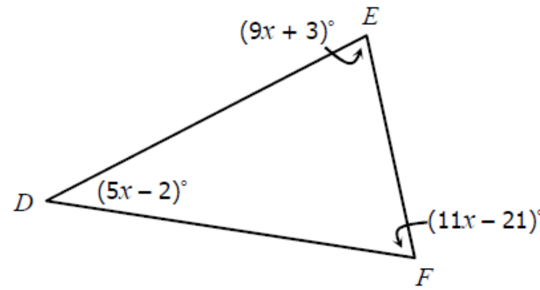
5) Find m<1



6) Find the m<1, m<2 & m<3.

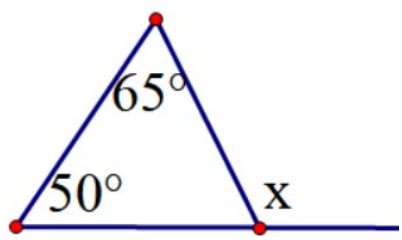


7) Find x

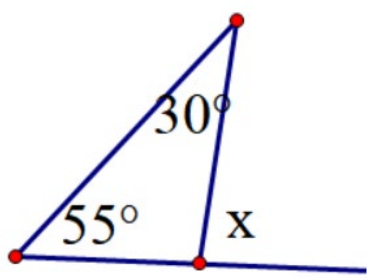


Find the measure of each exterior angle (x) in each example below:

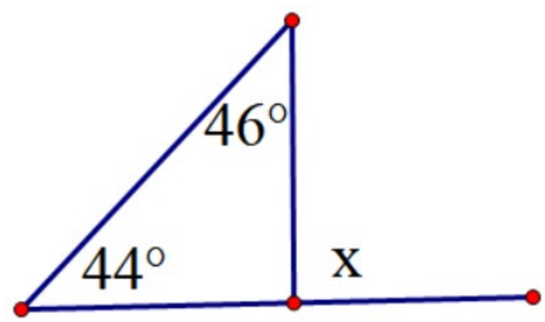
8)



9)



10)

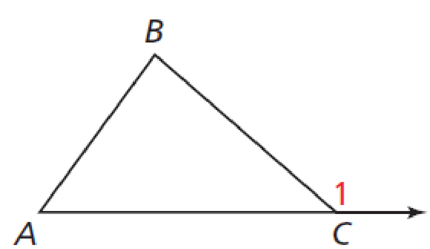


This leads us to our next theorem:

THEOREM

5.2 Exterior Angle Theorem

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.



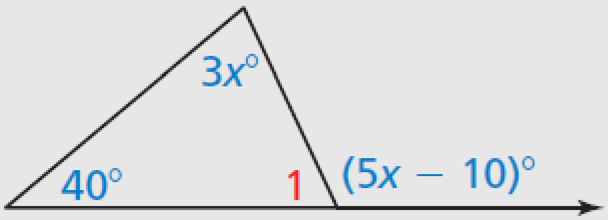
Prove this Theorem Exercise 42, page 235

$m\angle 1 = m\angle A + m\angle B$

Another few examples. Be sure to ANSWER THE QUESTION!

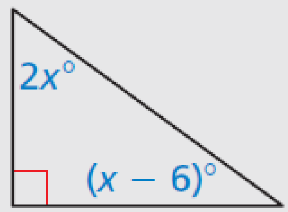
11)

Find $m\angle 1$.



12)

Find the measure of each acute angle.

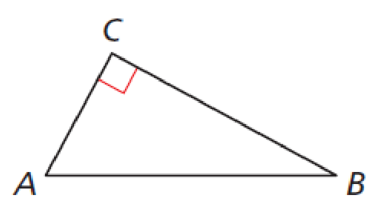


A **corollary to a theorem** is a statement that can be proved easily using the theorem. The corollary below follows from the Triangle Sum Theorem.

COROLLARY

5.1 Corollary to the Triangle Sum Theorem

The acute angles of a right triangle are complementary.



$m\angle A + m\angle B = 90°$

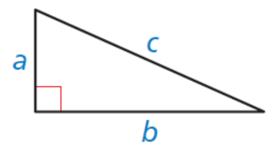
Prove this Corollary Exercise 41, page 235

Recall the Pythagorean Theorem introduced before:

THEOREM

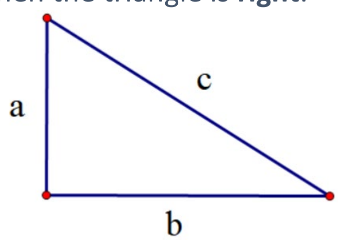
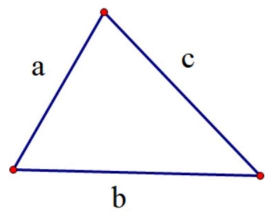
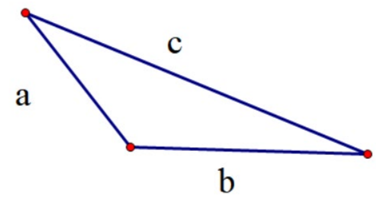
9.1 Pythagorean Theorem

In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

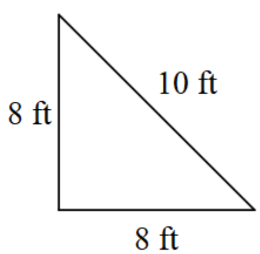
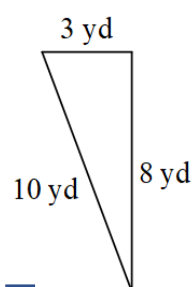
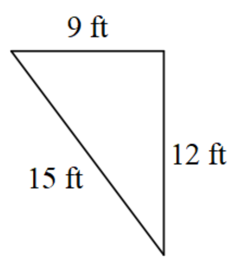


$c^2 = a^2 + b^2$

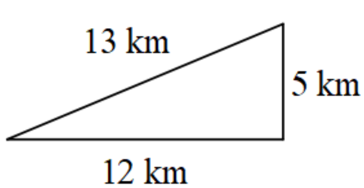
We can use the converse of this to classify a triangle by its angles given its side lengths.
Note: "c" must always be the longest side. The other two sides are "a" and "b" (interchangeable)

If $c^2 = a^2 + b^2$, then the triangle is **right**.  	If $c^2 < a^2 + b^2$, then the triangle is **acute**.  	If $c^2 > a^2 + b^2$, then the triangle is **obtuse**. 

Examples: Determine if the following triangles are right, acute or obtuse.
Show your work and fill in the box with "< or > or ="

1)  c^2 □ $a^2 + b^2$ 10^2 □ $8^2 + 8^2$ 100 □ 64 + 64 100 [<] 128 The triangle is acute.	2)  c^2 □ $a^2 + b^2$ 10^2 □ $3^2 + 8^2$ 100 □ 9 + 64 100 [>] 73 The triangle is obtuse.	3)  c^2 □ $a^2 + b^2$ 15^2 □ $9^2 + 12^2$ 225 □ 81 + 144 225 [=] 225 The triangle is right.

Your turn: Determine if the following triangles are right, acute or obtuse.

4)  	5) Side lengths are: 10 km, 12 km, 13 km	6) Side lengths are: 5 in, 10 in., 8 in.