Pg. 636 11-93 EOO

11-23 ALL ANGLES

11) $\sin x = \frac{\sqrt{3}}{2}$ Q.I $\rightarrow x = \frac{\pi}{3}$ Q.II $\rightarrow x = \frac{2\pi}{3}$

$$\boxed{x = \frac{\pi}{3} + 2\pi n, \; x = \frac{2\pi}{3} + 2\pi n; \; n = \text{INT}}$$

15) $\cos x = -\frac{1}{2}$ Q.III $\rightarrow x = \frac{2\pi}{3}$ Q.II $\rightarrow x = \frac{4\pi}{3}$

$$\boxed{x = \frac{2\pi}{3} + 2\pi n, \; x = \frac{4\pi}{3} + 2\pi n; \; n = \text{INT}}$$

19) $2\cos x + \sqrt{3} = 0$

$2\cos x = -\sqrt{3}$

$\cos x = -\frac{\sqrt{3}}{2}$ Q.III $\rightarrow x = \frac{5\pi}{6}$ Q.IV $\rightarrow x = \frac{7\pi}{6}$

$$\boxed{x = \frac{5\pi}{6} + 2\pi n, \; x = \frac{7\pi}{6} + 2\pi n; \; n = \text{INT}}$$

23) $3\sin\theta + 5 = -2\sin\theta$

$3\sin\theta + 2\sin\theta = -5$

$5\sin\theta = -5$

$\sin\theta = -1$

ONLY AT QUADRANTAL ANGLE, $\frac{3\pi}{2}$

$$\boxed{x = \frac{3\pi}{2} + 2\pi n; \; n = \text{INT}}$$

27-35 $[0, 2\pi)$ MULTIPLE ANGLES

27) $\cos 4x = -\frac{\sqrt{3}}{2}$ (QII, III)

$0 \le x < 2\pi$

$0 < 4x < 8\pi$ → 4 Positive Revolutions

$4x = \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{17\pi}{6}, \frac{19\pi}{6}, \frac{29\pi}{6}, \frac{31\pi}{6}, \frac{41\pi}{6}, \frac{43\pi}{6}$

$$\boxed{x = \frac{5\pi}{24}, \frac{7\pi}{24}, \frac{17\pi}{24}, \frac{19\pi}{24}, \frac{29\pi}{24}, \frac{31\pi}{24}, \frac{41\pi}{24}, \frac{43\pi}{24}}$$

31) $\tan\frac{x}{2} = \sqrt{3}$ Q.I and IV

$0 \le x < 2\pi$

$0 < \frac{x}{2} < \pi$ ← $\frac{x}{2}$ → Q I, II

Because it's positive then Q.I ONLY

$\frac{x}{2} = \frac{\pi}{3}, \frac{5\pi}{3}$

$\boxed{x = \frac{2\pi}{3}}, \cancel{\frac{10\pi}{3}}$ ← NOT in Quad I

35) $\sec\frac{3\theta}{2} = -2$ (QII, III)

$0 \le \theta < 2\pi$

$0 \le \frac{3\theta}{2} < 3\pi \leftarrow 1\frac{1}{2}$ Rev.

$\frac{3\theta}{2} = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}$

$\boxed{\theta = \frac{4\pi}{9}, \frac{8\pi}{9}, \frac{16\pi}{9}}$

39-51 Quadratics $[0, 2\pi)$

39) $2\sin^2 x - \sin x - 1 = 0$

$(2\sin x + 1)(\sin x - 1) = 0$

$2\sin x + 1 = 0 \qquad \sin x - 1 = 0$

$\sin x = -\frac{1}{2} \qquad \sin x = 1$

$x = \frac{7\pi}{6}, \frac{11\pi}{6} \qquad x = \frac{\pi}{2}$

$\boxed{x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}}$

43) $2\sin^2 x = \sin x + 3$

$2\sin^2 x - \sin x - 3 = 0$

$(2\sin x - 3)(\sin x + 1) = 0$

$2\sin x - 3 = 0 \qquad \sin x + 1 = 0$

$\sin x = \frac{3}{2} \qquad \sin x = -1$

$x = \frac{3\pi}{2}$

out of range > 1

$\boxed{x = \frac{3\pi}{2}}$

47) $4\cos^2 x - 1 = 0$

$4\cos^2 x = 1$

$\cos^2 x = 1/4$

$\cos x = \pm\frac{1}{2}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

47) $4\cos^2 x - 1 = 0$

$(2\cos x + 1)(2\cos x - 1) = 0$

(OR) $2\cos x + 1 = 0 \qquad 2\cos x - 1 = 0$

$\cos x = -1/2 \qquad \cos x = 1/2$

$\boxed{x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$

51) $\sec^2 x - 2 = 0$

$\sec^2 x = 2$

$\sec x = \pm\sqrt{2}$

$\therefore \cos x = \pm\frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

55, 61 $[0, 2\pi)$

55) $(2\cos x + \sqrt{3})(2\sin x + 1) = 0$

$2\cos x + \sqrt{3} = 0$ $\quad$ $2\sin x + 1 = 0$

$\cos x = -\frac{\sqrt{3}}{2}$ $\quad$ $\sin x = -\frac{1}{2}$

$x = \frac{5\pi}{6}, \frac{7\pi}{6}$ $\quad$ $x = \frac{7\pi}{6}, \frac{11\pi}{6}$

$x = \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

61) $\tan^2 x \cos x = \tan^2 x$

$\tan^2 x \cos x - \tan^2 x = 0$

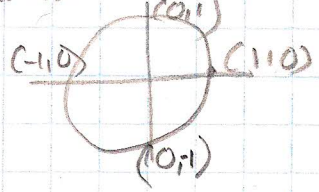
$\tan^2 x (\cos x - 1) = 0$

$\tan^2 x = 0$ $\quad$ $\cos x - 1 = 0$

$\tan x = 0$ $\quad$ $\cos x = 1$

$x = 0, \pi$ $\quad$ $x = 0$

Quadrantal Angles



$x = 0, \pi$

65 - 93 use IDENTITIES $[0, 2\pi)$

65) $\sin^2 x - 2\cos x - 2 = 0$

$(1 - \cos^2 x) - 2\cos x - 2 = 0$

$+\cos^2 x - 2\cos x - 1 = 0$

$\cos^2 x + 2\cos x + 1 = 0$

$(\cos x + 1)(\cos x + 1) = 0$

$\cos x + 1 = 0$

$\cos x = -1$ $\quad$ QUADRANTAL ANGLE $\quad$ $x = \pi$

69) $\sin 2x = \cos x$

$2\sin x\cos x = \cos x$

$2\sin x\cos x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

$\cos x = 0$ or $2\sin x - 1 = 0$

QUAD. I, II $\quad x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad \sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$\boxed{x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}}$

73) $\cos 2x + 5\cos x + 3 = 0$

$2\cos^2 x - 1 + 5\cos x + 3 = 0$

$2\cos^2 x + 5\cos x + 2 = 0$

$(2\cos x + 1)(\cos x + 2) = 0$

$2\cos x + 1 = 0$ or $\cos x + 2 = 0$

$\cos x = -\frac{1}{2}$ $\quad \cancel{\cos x = -2}$

Q. II, III $\quad$ OUT OF RANGE

$\boxed{x = \frac{2\pi}{3}, \frac{4\pi}{3}}$

77) $\sin x + \cos x = 1$ (Square both sides)

$(\sin x + \cos x)^2 = 1^2$

$\sin^2 x + 2\sin x\cos x + \cos^2 x = 1$

$\sin^2 x + \cos^2 x + 2\sin x\cos x = 1$

$1 + 2\sin x\cos x = 1$

$2\sin x\cos x = 0$

$\sin x\cos x = 0$

$\sin x = 0$ or $\cos x = 0$

(QUADRANTAL ANGLES)

$x = 0, \pi \quad x = \frac{\pi}{2}, \frac{3\pi}{2}$

However, Because we Squared BOTH Sides, we MUST Check ALL the ANSWERS.

IF $x = 0$ ✓ $\sin 0 + \cos 0 = 0 + 1 = 1$

IF $x = \frac{\pi}{2}$ ✓ $\sin\frac{\pi}{2} + \cos\frac{\pi}{2} = 1 + 0 = 1$

IF $x = \pi$ $\sin\pi + \cos\pi = 0 + (-1) = -1$ ✗

IF $x = \frac{3\pi}{2}$ $\sin\frac{3\pi}{2} + \cos\frac{3\pi}{2} = (-1) + 0 = -1$ ✗

} EXTRANEOUS SOLUTIONS

$\boxed{x = 0, \frac{\pi}{2}}$

81) $\sin 2x \cos x + \cos 2x \sin x = \frac{\sqrt{2}}{2}$ (Recognize this is the Sum formula for sin 5.2)

$\sin(2x+x) = \frac{\sqrt{2}}{2}$

$\sin 3x = \frac{\sqrt{2}}{2}$ Q.1, II ← $3x$ is a multiple angle.

$\therefore$ $0 \le x < 2\pi$

$3x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}, \frac{17\pi}{4}, \frac{19\pi}{4}$ $0 \le 3x < 6\pi$ ← 3 Revolutions

$x = \frac{\pi}{12}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{11\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}$