

NAME Keel

DATE

PER

(Rev. 4/7/2020)

Notes:

- Use the Law of Sines for oblique triangles, given AAS, ASA, and SSA. Remember that SSA is the ambiguous case, where the answer could be 0 triangle, 1 triangle, or 2 triangles.
- Use the Law of Cosines for oblique triangles, given SAS or SSS.
 - SAS: 1) Find the missing side.
2) Find the smallest angle (if not given).
3) Find the other angle using geometry.
 - SSS: 1) Find the largest angle
2) Use the Law of Sines to find any of the other 2 angles.
3) Find the third angle using geometry.

AREA OF TRIANGLES USING TRIGONOMETRY:

- If at least one angle is known: $Area = \frac{1}{2} (side)(side)(\sin \text{ of included angle})$.
If you are given SAS, then just plug in the numbers. If you are given AAS, or SSA, use LS to “force” SAS.

- If you are given SSS, then use Heron’s Formula:

$$Area = \sqrt{s(s - a)(s - b)(s - c)}$$

where s is the semiperimeter, and a, b, c, are the sides of the triangle.



SHOW ALL WORK ON A SEPARATE SHEET OF PAPER. DO NOT OMIT ANY STEP.

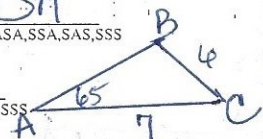
I. In problems 1-4, given oblique triangle ABC, find a side (to the nearest tenth) or an angle (to the nearest degree). Draw each triangle and determine if it is AAS, ASA, SSA, SAS, or SSS. Write it on the line.

1) $\angle A = 32^\circ, \angle B = 41^\circ, a = 20$. Find side c. AAS
AAS, ASA, SSA, SAS, SSS

2) $a = 26, b = 41, c = 37$. Find $\angle A$. SSS
AAS, ASA, SSA, SAS, SSS

3) $\angle C = 15^\circ, a = 30, \angle B = 73^\circ$. Find side c. ASA
AAS, ASA, SSA, SAS, SSS

4) $a = 6, b = 7, \angle A = 65^\circ$. Find $\angle B$. SSA
AAS, ASA, SSA, SAS, SSS



$\frac{6}{\sin 65} = \frac{7}{\sin B}$
 $\sin B = \frac{7 \sin 65}{6}$
 $\sin B \approx 1.0574$
out of range [-1, 1]!
no Δ !

II. In problems 5-8, solve triangle ABC. If no triangle exist, explain in detail. Round sides to the nearest tenth and angles to the nearest degree. Draw each triangle and determine if it is AAS, ASA, SSA, SAS, or SSS. Write it on the line.

5) $a = 5, b = 3, c = 7$ SSS
AAS, ASA, SSA, SAS, SSS

6) $\angle B = 110^\circ, a = 10, c = 16$ SAS
AAS, ASA, SSA, SAS, SSS

7) $a = 26, b = 30, \angle A = 52^\circ$ SSA
AAS, ASA, SSA, SAS, SSS

8) $\angle A = 46^\circ, a = 86, c = 65$ SSA
AAS, ASA, SSA, SAS, SSS

III. In problems 9 and 10, find the area of the triangle. Round the answer to the nearest unit.

9) $\angle C = 36^\circ$, $a = 5$ ft, $b = 7$ ft.

10) $a = 7$ m, $b = 9$ m, $c = 12$ m

IV. IV: Word Problems

11) Two trucks leave a city at the same time and travel along straight highways that differ in direction by 75° . One truck averages 80 miles per hour and the other averages 70 miles per hour.

a) How far apart will the trucks be after 30 minutes? Round to the nearest tenth of a mile.

b) How far apart will the trucks be after 3 hours? Round to the nearest tenth of a mile.

12) The Leaning Tower of Pisa leans at an angle of about 85° . 180ft from the base of the tower, the angle of elevation to the top is 65° . Find the distance to the nearest tenth of a foot, from the base to the top of the tower.

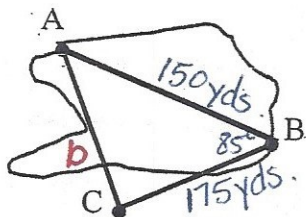
13) A surveyor needs to determine the distance between two points that lie on opposite banks of a river. 500 yards are measured along one bank. The angles from each end of this line segment (AB) to a point on the opposite bank (C) are 70° from point A and 47° from point B. Find the distance between point B and C to the nearest tenth of a yard.

14) Use the figure to find the distance across the lake from A to C, to the nearest yard. $AB = 150$ yds, $BC = 175$ yds, and $m\angle B = 85^\circ$

$$b^2 = 175^2 + 150^2 - 2(175)(150)\cos 85^\circ$$

$$=$$

(SAS)
- $\angle C$



15) Use the same drawing above (problem 14). If $AB = 11$ miles, $BC = 13$ miles, and $AC = 19$ miles, what is the measure of $\angle A$?

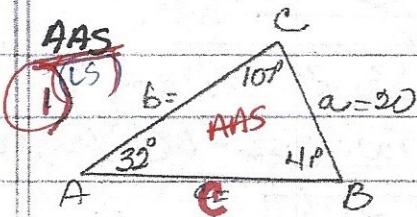
16) A plane leaves airport A and travels 600 miles to airport B on a bearing of $N45^\circ E$. The plane later leaves airport B and travels to airport C 450 miles on a bearing of $S85^\circ E$. Find the distance from airport A to airport C to the nearest tenth.

1) $C \approx 36.1$	7) $B =$ $C =$ $c =$	11) a) $c \approx 45.8$ mi b) $c \approx 275.0$ mi
2) $A \approx 38^\circ$	8) $B \approx 101^\circ$ (1A) $C = 33^\circ$ $b \approx 117.4$	12) $a \approx 326.3$ ft.
3) $C \approx 7.8$		13) $a \approx 527.3$ yds.
4) $\angle B$ does NOT EXIST.		14) 220 yds.
5) $A = 38^\circ, B = 22^\circ, C = 120^\circ$	9) Area ≈ 10 ft ²	15) $A \approx 42^\circ$
6) $A = 26^\circ, C = 44^\circ, b = 21.6$	10) Area ≈ 31 m ²	16) $b \approx 953.7$ mi

* Write sentences under each word problem.

①

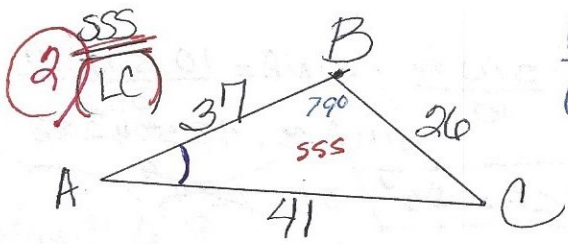
6.1-6.2 PRACTICE TEST WORK/ANSWERS



① $m\angle C = 107^\circ$

② $\frac{\sin 32}{20} = \frac{\sin 107}{c}$

$c = \frac{20 \sin 107}{\sin 32}$; $c \approx 36.1$

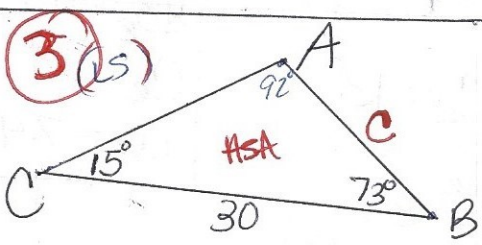


FIND Largest & First! ($\angle B$)

① $b^2 = a^2 + c^2 - 2ac \cos B$
 $41^2 = 26^2 + 37^2 - 2(26)(37) \cos B$
 $1681 = 676 + 1369 - 1924 \cos B$
 $1681 = 2048 - 1924 \cos B$
 $-367 = -1924 \cos B$
 $\cos B = \frac{367}{1924}$

$B \approx 79^\circ$

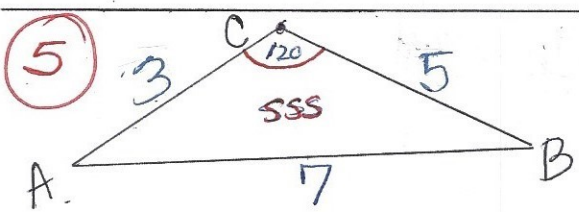
② $\frac{41}{\sin 79} = \frac{26}{\sin A}$
 $\sin A = \frac{26 \sin 79}{41}$
 $\sin A \approx .6225$
 $A \approx 38^\circ$



① $A = 92^\circ$
 ② $\frac{30}{\sin 92} = \frac{c}{\sin 15}$
 $c = \frac{30 \sin 15}{\sin 92}$

#4 see front of WS

$c \approx 7.8 \text{ units}$

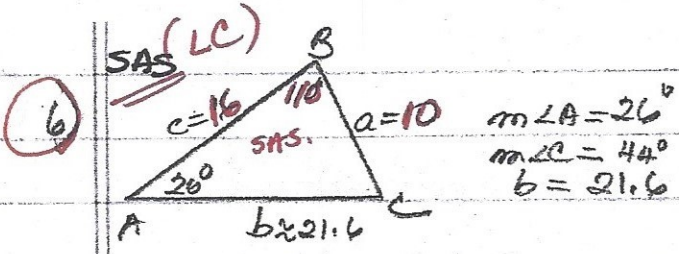


① $c^2 = a^2 + b^2 - 2ab \cos C$
 $7^2 = 3^2 + 5^2 - 2(5)(3) \cos C$
 $49 = 9 + 25 - 30 \cos C$
 $49 = 34 - 30 \cos C$
 $\frac{15}{-30} = -30 \cos C$
 $\cos C = -\frac{1}{2}$ ← (ref 60)

② $\frac{7}{\sin 120} = \frac{3}{\sin B}$
 $\sin B = \frac{3 \sin 120}{7}$
 $B \approx 22^\circ$

$C = 120$

$A = 38^\circ$



2

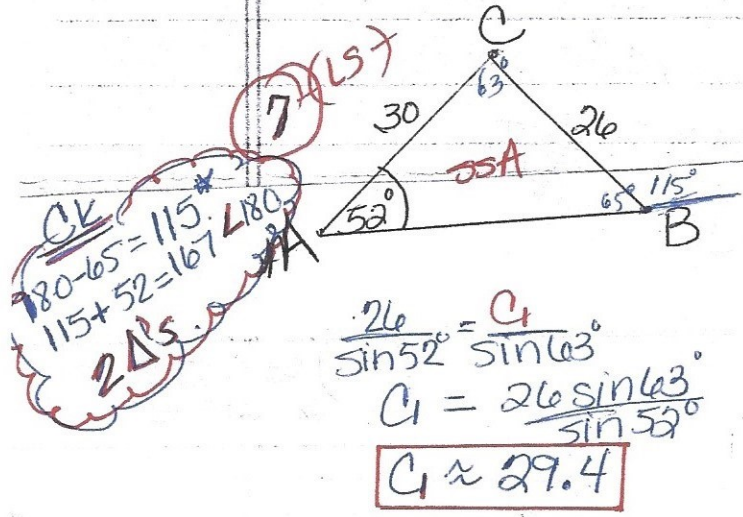
$b \approx 21.6$
 $A \approx 26^\circ$
 $C \approx 44^\circ$

① Find missing side: $b^2 = a^2 + c^2 - 2ac \cos B$
 $b^2 = 100 + 256 - 2(10)(16) \cos 110$

$b \approx 21.6$

② Find smallest \angle : ($\angle A$)
 $\frac{\sin 110}{21.6} = \frac{\sin A}{10}$; $\sin A = \frac{10 \sin 110}{21.6}$
 $\sin A \approx .43504288$

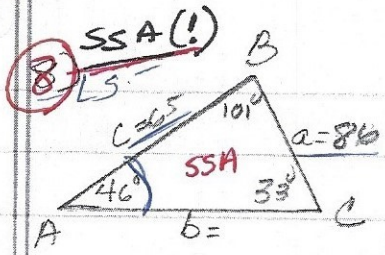
$\angle A \approx 26^\circ$ or 154°
 Δ is obtuse!
 $B = 110$



$\frac{26}{\sin 52} = \frac{30}{\sin B}$
 $\sin B = \frac{30 \sin 52}{26}$
 $B_1 \approx 65^\circ$ $C_1 \approx 63^\circ$
 $B_2 = 115^\circ$ $C_2 = 13^\circ$
 $\frac{26}{\sin 52} = \frac{c_2}{\sin 13}$
 $c_2 = \frac{26 \sin 13}{\sin 52}$
 $c_2 \approx 7.4$

$B_1 = 65^\circ$
 $C_1 = 63^\circ$
 $c_1 = 29.4$
 $B_2 = 115^\circ$
 $C_2 = 13^\circ$
 $c_2 = 7.4$

3



$$B \approx 101^\circ$$

$$C \approx 33^\circ$$

$$b \approx 117.4$$

$$\frac{\sin 46}{86} = \frac{\sin C}{65}$$

$$\sin C = \frac{65 \sin 46}{86}$$

$$\frac{\sin 46}{86} = \frac{\sin 101}{b}$$

$$b = \frac{86 \sin 101}{\sin 46}$$

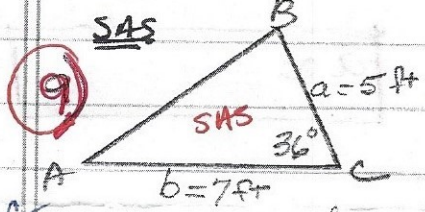
$$b \approx 117.4$$

$$\sin C = .5436870584$$

$$C = 33^\circ \text{ OR } 147^\circ$$

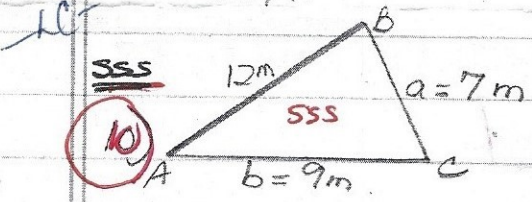
CK: $180 - 33 = 147$
 $+ 46$
 $193 > 180^\circ$

1 Δ



$$\text{Area} = \frac{1}{2} (5)(7) \sin 36^\circ$$

$$\text{Area} \approx 10 \text{ ft}^2$$



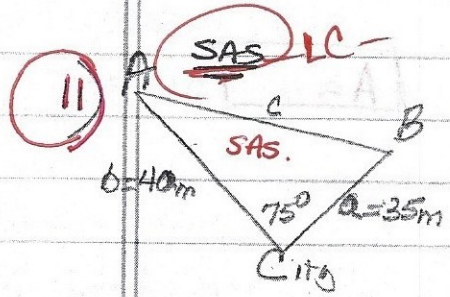
$$\text{Area} = \sqrt{14(14-12)(14-7)(14-9)}$$

$$= \sqrt{14(2)(7)(5)}$$

$$A \approx 31 \text{ m}^2$$

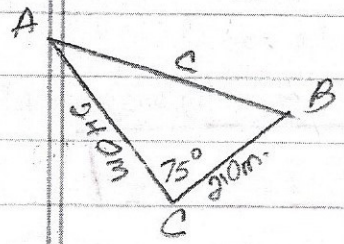
$$P = 28 \text{ m}$$

$$S = 14 \text{ m}$$



a) $c^2 = 35^2 + 40^2 - 2(35)(40) \cos 75$

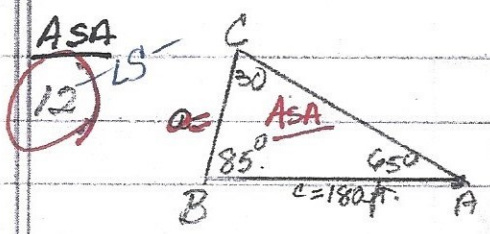
$$c \approx 45.8 \text{ mi}$$



b) $c^2 = 240^2 + 210^2 - 2(240)(210) \cos 75$

$$c \approx 275.0 \text{ mi}$$

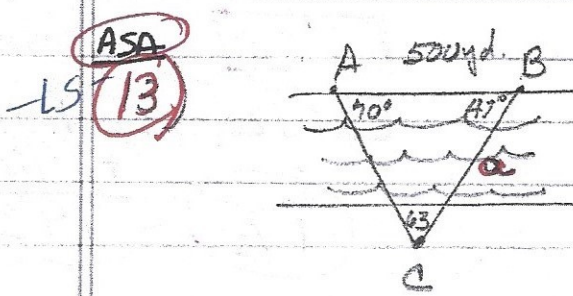
4



$m\angle C = 30^\circ$ $\frac{\sin 30}{180} = \frac{\sin 65}{a}$

$a = \frac{180 \sin 65}{\sin 30}$; $a \approx 326.3$

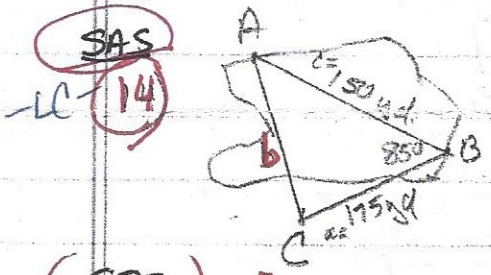
326.3 ft



$m\angle C = 63^\circ$ $\frac{\sin 63}{500} = \frac{\sin 70}{a}$

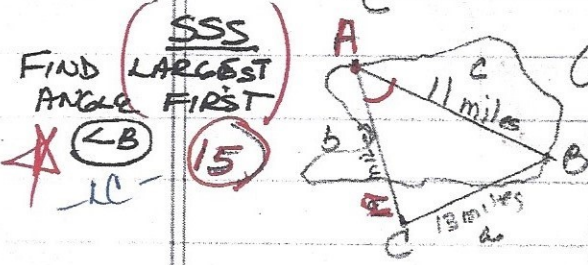
$a = \frac{500 \sin 70}{\sin 63}$

a ≈ 527.3 yds



$b^2 = 175^2 + 150^2 - 2(175)(150) \cos 85$

b ≈ 220 yd



$b^2 = a^2 + c^2 - 2ac \cos B$

$19^2 = 13^2 + 11^2 - 2(13)(11) \cos B$

$361 = 169 + 121 - 286 \cos B$

$361 = 290 - 286 \cos B$

$71 = -286 \cos B$

$\cos B = -.2482517483$

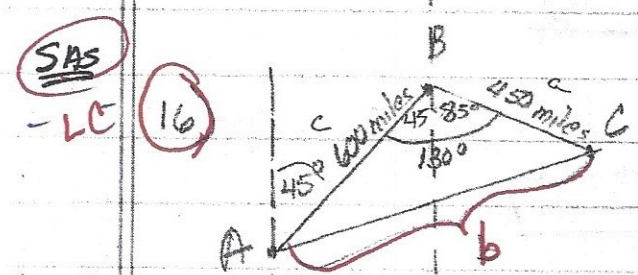
Ref $\angle = 76^\circ$ $B = 104^\circ$

$\frac{\sin 104}{19} = \frac{\sin A}{13}$

$\sin A = 13 \frac{\sin 104}{19}$

$\sin A = .6638865496$

A ≈ 42° or ~~138°~~



$b^2 = a^2 + c^2 - 2ac \cos B$

$b^2 = 450^2 + 600^2 - 2(450)(600) \cos 130^\circ$

b ≈ 953.7 miles

Those angles are alternate interior angles
A1A → ≅