Sec (7.3) Partial Fractions

<u>Partial Fraction Decomposition</u> is the "Reverse" process of writing a sum (+) or difference (-) of rational expressions as a single rational expression.

<u>rational expressions</u> \rightarrow The quotient of two polynomials. Ex $\rightarrow \frac{2x-3}{(x^2-64)}$

- In short when we (+) or (-) fractions we find the LCD and "combine" into one fraction.
- To decompose a fraction is to "separate"/ "decompose" one fraction back into a (+) or
 (-) or two or more fractions.

In order to decompose [used for integration in Calc.] a few things must be checked.

- 1) Degree of numerator must be less than degree of denominator. [if its an *improper fraction, you must reduce by long division → will learn in Calc.]
- 2) Check the "Type" of decomposition needed. i.e. [Because of Distance Learning we will only do (a-c)]
 - a) <mark>Linear Factors</mark>

b) Repeated Linear Factors

- c) Prime-Quadratic Factors [if they factor, they become linear]
- d) Repeated Quadratic Factors
- e) Improper Fractions*

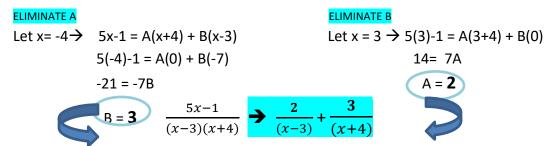
Decomposing into Partial Fractions

<u>STEPS:</u>

- 1) Factor^{*} each denominator
- 2) Express the problem as a sum of two fractions with Numerators "A" or "B" use the "factors" for denominators*
- 3) Eliminate denominators by multiplying each term by the LCD
- 4) Eliminate "A" by letting the parenthesis next to "A" =0. Solve for "B"
- 5) Eliminate "B" by letting the parenthesis next to "B" =0. Solve for "A" **Sometimes a system of Linear equations is needed to find the <u>values</u>**
- 6) Substitute the *values* found for "A" and "B" back in *Step 2*
- 7) **Check** to see if the sum of the two partial fractions equals the original fraction.

Example: 1 [Linear Factors] Find the partial fraction decomposition of $\frac{5x-1}{(x-3)(x+4)}$

$$\frac{5x-1}{(x-3)(x+4)} = \frac{A}{(x-3)} + \frac{B}{(x+4)} \qquad \Rightarrow Mult. \ ea. \ term \ by \ \mathsf{LCD} \rightarrow [((x-3)(x+4))]$$



Example: 2 [repeated Linear factors] Find the partial fraction decomposition of $\frac{x+2}{x(x-1)^2}$

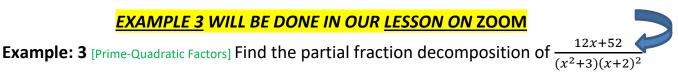
$$\frac{x+2}{x(x-1)^2} = \frac{A}{x} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2} \quad \Rightarrow Mult. \ ea. \ term \ by \ LCD \rightarrow [x(x-1)^2]$$

 $X + 2 = A(x - 1)^2 + B x(x - 1) + Cx$ $= A(x^2 - 2x + 1) + B(x^2 - x) + Cx$ = $Ax^2 - A2x + A + Bx^2 - Bx + Cx$ \rightarrow (group) by var & degree $X + 2 = (A + B)x^2 + (-2A - B + C)x + A$ \rightarrow Use the coefficients of x + 2 to set up a <u>system</u> & find values

System →
$$\begin{cases} A + B = 0 \\ -2A - B + C = 1 \\ A = 2 \end{cases}$$

A + B = 0
-2A - B + C = 1
2 + B = 0
-2(2) - (-2) + C = 1
B = -2
-4 + 2 + C = 1
-2 + C = 1
-2 + C = 1
C = 3 \end{cases}
$$\frac{x+2}{x(x-1)^2} \Rightarrow \frac{2}{x} - \frac{2}{(x-1)} + \frac{3}{(x-1)^2}$$

<u>EXAMPLE 3</u> WILL BE DONE IN OUR <u>LESSON ON ZO</u>OM



SEPARATE FILE HAS WORK***