Sec (7.3) Partial Fractions

Partial Fraction Decomposition is the "Reverse" process of writing a sum (+) or difference (-) of rational expressions as a single rational expression.

rational expressions→The quotient of two polynomials. Ex→ $\frac{2x-3}{(x^2-64)}$

- In short when we (+) or (-) fractions we find the LCD and "combine" into one fraction.
- To decompose a fraction is to "separate"/ "decompose" one fraction back into a (+) or (-) or two **or** more fractions.

In order to decompose **[used for integration in Calc.]** a few things must be checked.

1) Degree of numerator must be **less than** degree of denominator. [if its an *improper fraction, you must reduce by long division→ ***will learn in Calc***.]
2) Check the "*Type*" of decomposition needed. i.e. ***[Because of Distance Learning –we will only do (a-c)]***

   a) Linear Factors

   b) Repeated Linear Factors

   c) Prime-Quadratic Factors *[if they factor, they become linear]*

   d) Repeated Quadratic Factors

   e) Improper Fractions*

Decomposing into Partial Fractions→

STEPS:

1) Factor* each denominator
2) Express the problem as a **sum** of two fractions with Numerators **"A"** or **"B"** use the "factors" for denominators*
3) Eliminate denominators by multiplying each term by the LCD
4) Eliminate "A" by letting the parenthesis next to "A" =0. Solve for "B"
5) Eliminate "B" by letting the parenthesis next to "B" =0. Solve for "A"

   ****Sometimes a system of Linear equations is needed to find the *values*****

6) Substitute the ***values*** found for "A" and "B" back in ***Step 2***
7) **Check** to see if the sum of the two partial fractions equals the original fraction.

Example: 1 **[Linear Factors]** Find the partial fraction decomposition of $\frac{5x-1}{(x-3)(x+4)}$

$\frac{5x-1}{(x-3)(x+4)} = \frac{A}{(x-3)} + \frac{B}{(x+4)}$ → *Mult. ea. term by* **LCD**→ $[((x-3)(x+4)]$

ELIMINATE A

Let x= -4→ 5x-1 = A(x+4) + B(x-3)

5(-4)-1 = A(0) + B(-7)

-21 = -7B

B = **3**

ELIMINATE B

Let x = 3 → 5(3)-1 = A(3+4) + B(0)

14= 7A

A = **2**

$\frac{5x-1}{(x-3)(x+4)}$ → $\frac{2}{(x-3)} + \frac{3}{(x+4)}$

Example: 2 [repeated Linear factors] Find the partial fraction decomposition of $\frac{x+2}{x(x-1)^2}$

$\frac{x+2}{x(x-1)^2} = \frac{A}{x} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2}$ ➔*Mult. ea. term by* **LCD**→ $[x(x-1)^2]$

$X + 2 = A(x-1)^2 + B\,x(x-1) + Cx$

$= A(x^2 - 2x + 1) + B(x^2 - x) + Cx$

$= Ax^2 - A2x + A + Bx^2 - Bx + Cx$ ➔ (group) by *var* & degree

$X + 2 = (A + B)x^2 + (-2A - B + C)x + A$ ➔ Use the coefficients of x + 2 to *set up a* ***system*** & *find values*

System → $\begin{cases} A + B = 0 \\ -2A - B + C = 1 \\ A = 2 \end{cases}$

A + B = 0	-2A – B + C = 1
2 + B = 0	-2(2) – (-2) + C = 1
B= –2	-4 + 2 + C = 1
	-2 + C = 1
	C = 3

$\frac{x+2}{x(x-1)^2}$ → ➔ $\frac{2}{x} - \frac{2}{(x-1)} + \frac{3}{(x-1)^2}$

EXAMPLE 3 WILL BE DONE IN OUR LESSON ON ZOOM

Example: 3 [Prime-Quadratic Factors] Find the partial fraction decomposition of $\frac{12x+52}{(x^2+3)(x+2)^2}$

SEPARATE FILE HAS WORK***