**Sec (7.3) Partial Fractions 4/20/20**

*Partial Fraction Decomposition* is the “Reverse” process of writing a sum (+) or difference (-) of rational expressions as a single rational expression.

*rational expressions*🡪The quotient of two polynomials. Ex🡪 $\frac{2x-3}{(x^{2}-64)}$

* In short when we (+) or (-) fractions we find the LCD and “combine” into one fraction.
* To decompose a fraction is to “separate”/ “decompose” one fraction back into a (+) or (-) or two **or** more fractions.

*In order to decompose* **[used for integration in Calc.]** a few things must be checked.

1. Degree of numerator must be **less than** degree of denominator. [if its an **\***improper fraction, you must reduce by long division🡪 ***will learn in Calc***.]
2. Check the “*Type*” of decomposition needed. i.e. ***[Because of Distance Learning –we will only do (a-c)]***

a) Linear Factors

b) Repeated Linear Factors

c) Prime-Quadratic Factors *[if they factor, they become linear]*

d) Repeated Quadratic Factors

e) Improper Fractions**\***

**Decomposing into Partial Fractions🡺**

***STEPS:***

1. Factor\* each denominator
2. Express the problem as a **sum** of two fractions with Numerators **“A”** or **“B”** use the “factors” for denominators\*
3. Eliminate denominators by multiplying each term by the LCD
4. Eliminate “A” by letting the parenthesis next to “A” =0. Solve for “B”
5. Eliminate “B” by letting the parenthesis next to “B” =0. Solve for “A”

**\*\*Sometimes a system of Linear equations is needed to find the *values*\*\***

1. Substitute the ***values*** found for “A” and “B” back in ***Step 2***
2. **Check** to see if the sum of the two partial fractions equals the original fraction.

**Example: 1** **[Linear Factors]** Find the partial fraction decomposition of $\frac{5x-1}{(x-3)(x+4)}$

$\frac{5x-1}{(x-3)(x+4)}$ =$\frac{A}{(x-3)}$ + $\frac{B}{(x+4)}$ 🡺*Mult. ea. term by* **LCD🡪** [($(x-3)(x+4)$]

ELIMINATE A ELIMINATE B

Let x= -4🡪 5x-1 = A(x+4) + B(x-3) Let x = 3 🡪 5(3)-1 = A(3+4) + B(0)

 5(-4)-1 = A(0) + B(-7) 14= 7A

 -21 = -7B A = **2**

 B = **3** $\frac{5x-1}{(x-3)(x+4)}$ 🡺 $\frac{2}{(x-3)}$ + $\frac{3}{(x+4)}$

**Example: 2** **[repeated Linear factors]** Find the partial fraction decomposition of $\frac{x+2}{x\left(x-1\right)^{2}}$

$\frac{x+2}{x\left(x-1\right)^{2}}$ = $\frac{A}{x}$ + $\frac{B}{(x-1)}$ + $\frac{C}{\left(x-1\right)^{2}}$ 🡺*Mult. ea. term by* **LCD🡪** [x$\left(x-1\right)^{2}$]

X + 2 = A$\left(x-1\right)^{2}$ + B$ x(x-1)$ + C*x*

 = A(x2 – 2x + 1) + B(x2 – x) + Cx

 = Ax2 – A2**x** + A + Bx2 – B**x** + C**x**  🡺 (group) by *var* & degree

X + 2 = (A + B)**x2** + (-2A – B + C)**x** + A 🡺 Use the coefficients of x + 2 to *set up a* ***system*** & *find values*

***System*** 🡪 $\left\{\begin{array}{c}A+B=0 \\-2A-B+C=1\\A=2\end{array}\right.$

 A + B = 0 -2A – B + C = 1

 2 + B = 0 -2(2) – (-2) + C = 1

 B= –2 -4 + 2 + C = 1

 -2 + C = 1

 C = 3

$\frac{x+2}{x\left(x-1\right)^{2}}$ 🡺 🡺 $\frac{2}{x}$ – $\frac{2}{(x-1)}$ +$\frac{3}{\left(x-1\right)^{2}}$

 ***EXAMPLE 3 WILL BE DONE IN OUR LESSON ON* ZOOM**

**Example: 3** [Prime-Quadratic Factors] Find the partial fraction decomposition of $\frac{12x+52}{(x^{2}+3)\left(x+2\right)^{2}}$

SEPARATE FILE HAS WORK\*\*\*