Example #3 [Prime Quadratic & Repeated Linear Factors]

Find the partial fraction decomposition of

$$\frac{12x+52}{(x+2)^2(x^2+3)}$$

* LCM ⇒ $(x+2)^2(x^2+3)$

* Get Rid of Fraction. Use LCD.

$$\frac{12x+52}{(x+2)^2(x^2+3)} = \underbrace{\frac{A}{(x+2)^2}}_{\text{linear}} + \underbrace{\frac{B}{(x+2)}}_{\text{linear}} + \underbrace{\frac{Cx+D}{(x^2+3)}}_{\text{Prime Quadratic}}$$

* $12x+52 = A(x^2+3) + B[(x^2+3)(x+2)] + (Cx+D)(x+2)^2$

$B(x^3+2x^2+3x+6)$ $\quad (Cx+D)(x^2+4x+4)$

$$= Ax^2+3A+Bx^3+2Bx^2+3Bx+6B+Cx^3+4Cx^2+4Cx+Dx^2+4Dx+4D$$

Group by Var & degree:

$$12x+52 = (B+C)x^3+(A+2B+4C+D)x^2+(3B+4C+4D)x+3A+6B+4D$$

* use Coeff. to set Up System:

$$\begin{cases} B+C=0 \\ A+2B+4C+D=0 \\ 3B+4C+4D=12 \\ 3A+6B+4D=52 \end{cases}$$

M(-3) (2nd → 4th); M(-3) (1st → 3rd)

I

M-3 2nd & 4th

$$\begin{aligned} -3A-6B-12C-3D &= 0 \\ 3A+6B+4D &= 52 \\ \hline -12C+D &= 52 \end{aligned}$$

M-3 1st & 3rd

$$\begin{aligned} 3B+4C+4D &= 12 \\ -3B-3C &= 0 \\ \hline C+4D &= 12 \end{aligned}$$

II

$-12C + D = 52$

$C + 4D = 12 \longrightarrow M(12)$

$-12C + D = 52$

$12C + 48D = 144$

$49D = 196$

$D = 4$

$C + 4(4) = 12$

$C + 16 = 12$

$-16 \quad -16$

$C = -4$

1st eq.

$B + C = 0$

$B - 4 = 0$

$B = 4$

2nd eq (for A)

$A + 2B + 4C + D = 0$

$A + 2(4) + 4(-4) + 4 = 0$

$A + 8 - 16 + 4 = 0$

$A - 8 + 4 = 0$

$A - 4 = 0$

$A = 4$

ANS

$$\frac{12x+52}{(x+2)^2(x^2+3)} \rightarrow \frac{4}{(x+2)^2} + \frac{4}{(x+2)} + \frac{-4x+4}{(x^2+3)}$$