Section 8.4 ➔ Multiplicative Inverses of Matrices & *Matrix Equations*

In mathematics, the "Mult. Inverse" property states that it is a number **[reciprocal]** whose product = 1. ***i.e.*** the *Mult. Inv*. of ***a*** is $\frac{1}{a}$ since ***a*** ($\frac{1}{a}$) = 1.

In "*the world of 'MATRICES'*" the only operation that we cannot perform IS *division*, and the number **1** is represented by the "*I*-dentity Matrix" ➔ $I_n = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ where ***n*** is a 2X2 or can be a 3X3 sq. matrix $I_n = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, etc… So, to compensate for not having division, WE….

MULTIPLY **BY THE "Mult. INVERSE OF THE MATRIX"**

The ***inverse notation*** in mathematics is $\mathbf{x^{-1}}$, so the *inverse of matrix* A is denoted by A^{-1}.

- To find the "**Mult. Inv." of a square matrix** we use the ***definition*** **[p.922]**➔

❖ **2X2** ➔ **IF A =** $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$**, then** $A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ **where |A|**→***det(A)*** [the diff. of the X-product]

❖ **3X3** ➔ **use the Calculator. [Or it will be given.]** ∴ To prove two matrices are inverses,

If $AA^{-1} = I$ and $A^{-1}A = I$ then you just proved that the 2 matrices are inverse, and are "*invertible.*" → a matrix who has an inverse.

Show Work for **|A|** below

Ex -1) Find the Inverse of A, given: $A = \begin{bmatrix} -1 & -1 \\ -3 & 2 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ ➔ Step 1→ Find **|A| = -5** **[**If the det(A) = 0, the matrix has no inverse & is ***"Singular"***]

$= \frac{1}{-5}\begin{bmatrix} 2 & 1 \\ 3 & -1 \end{bmatrix}$ ← Step 2 *Switch* a_{11} and a_{22} **&** *negate* the *other diag. entries*.

$A^{-1} = \begin{bmatrix} \frac{-2}{5} & \frac{-1}{5} \\ \frac{-3}{5} & \frac{1}{5} \end{bmatrix}$ ← Step 3 **[answer]** mult. By the scalor

Check: If $A^{-1}A = I$ if the product = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then your work is correct.

$$\begin{bmatrix} \frac{-2}{5} & \frac{-1}{5} \\ \frac{-3}{5} & \frac{1}{5} \end{bmatrix}\begin{bmatrix} -1 & -1 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

MATRIX EQUATIONS → You are only responsible for →

- **The *set up*** of a 2X2 and 3X3 matrix eq. for a system.
- To **SOLVE a 2X2** matrix eq.—to solve a 3X3 you need a calculator.

STEPS To solve a *system* of linear equations using Matrix equations →

1) Set up ***3-Matrix equations*** using the *system* given.

   ****setup →**

   - *Coefficient* Matrix =[will **always** by a *square matrix* i.e. 2X2 or 3X3]
   - *Variable* Matrix =[will **always** by a column matrix → 2X1 or 3X1]
   - *Constant* Matrix =[will **always** by a column matrix → 2X1 or 3X1]

2) Find the **inverse** of the *Coefficient Matrix* **[2X2 use def/3X3 use calc]**
3) **Multiply** the "Constant" Matrix by the Inverse matrix **[from step 2]**
4) Write your solution as an <u>*ordered pair* 2X2</u> **or** <u>*ordered triple* 3X3</u>

Algebraic Representation→ If Ax = B then mult. *Both sides* by the "*Mult. Inv. of A*" $\mathbf{A^{-1}}$

$\frac{1}{A}$Ax = $\frac{1}{A}$B ***so*** $x = \frac{B}{A}$ → Make sure you place A^{-1} **in front** of constant Matrix.

<u>Ex of a Matrix Eq and Solution following steps/setup above</u> ** →

Ex1) 2X2 → **Solve** the system by *using Matrix Equations.*

[*hint:* (1)-set up eq. (2)-find inverse (3)-mult. both sides by inverse to solve]

$$\begin{cases} 4x - 7y = -3 \\ 2x - 3y = 1 \end{cases}$$

<u>Show Work for ***inverse*** below</u>

MATRIX EQUATION

$$\begin{bmatrix} 4 & -7 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

Coeff. Matrix 2X2   *Var.* 2X1   *Constants* 2X1

Inverse of Coeff. Matrix

$$\begin{bmatrix} \frac{-3}{2} & \frac{7}{2} \\ -1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} \frac{-3}{2} & \frac{7}{2} \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 4 & -7 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{-3}{2} & \frac{7}{2} \\ -1 & 2 \end{bmatrix} \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

<u>product yields→ ***I***</u>   <u>product yields ***Ans***</u>.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \end{bmatrix}$$

∴ (8, 5) is the solution to the system.

Ex 2) Write a ***Matrix equation*** for the following 3X3 system.

$$\begin{cases} 2x + 2y + 3z = 10 \\ 4x - y + z = -5 \\ 5x - 2y + 6z = 1 \end{cases}$$