Section 8.5 → Determinants & Cramer's Rule

Determinants→

Def.--> A Determinant is a ℝ ***number*** associated with a square matrix.

- While we find the value of the ***det***. of a square matrix, the *symbol* is different.
- ***Matrices*** are in brackets **[a]** && ***determinant***s are inside **| |** lines….

i.e.

A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ *Operations* w/matrices yield another matrix.

det(A) = $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ To ***evaluate*** the det. Of a matrix yields a ℝ ***number***.

To *evaluate* a **2X2** matrix we use the following procedure/RULE:

det(A) = $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ → (ad – bc) **[diff. of the cross product of the entries starting with a_{ij}]**

= (ad – bc)

Ex 1) **Evaluate** the determinant of matrix A. A = $\begin{bmatrix} 4 & 3 \\ -5 & -8 \end{bmatrix}$

det (A) = $\begin{vmatrix} 4 & 3 \\ -5 & -8 \end{vmatrix}$ → 4(-8) – (-5)(3)

-32 + 15

det(A) = – 17

A **3X3** matrix is *evaluated* by→ 1>Diagonal Method

2> A Calculator

3> Expansion by minors **[p.939—not doing this method]**

Ex 2) Evaluate: $\begin{vmatrix} 6 & 4 & 0 \\ -3 & -5 & 3 \\ 1 & 2 & 0 \end{vmatrix}$ we will use the **diagonal method** & then the calculator to check.

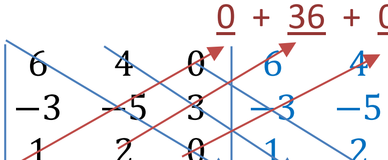
Process→ 1→ "take out" [write] first two columns and place outside of determinant. [blue below]

2 → Mult. entries in diagonal and Add the products together **[see below]**

3→ Subtract sum of products→ ***Bottom value*** **–** ***Top value***

Top value

0 + 36 + 0 = 36



12 – 36 = – 24

0 + 12 + 0 = 12

Bottom value

Cramer's Rule ➔ A rule used to solve systems of linear equations using *determinants*.

- You can use it to solve any type of system. ***i.e.*** 2X2 or 3X3, etc...
- Since we use **determinants** the "matrix" setup of the equations must be a square matrix.
- There is a Major determinant: set up with the Coeff. Of the variables.
- There are Minor determinants: this depends of the number of vars/eq. in system.

RULEs→

1) Set up & evaluate the ***Major det. D*** first. If it equals ***0***, there is ***no solution***.
2) If Major ≠ 0, then set up & evaluate all the ***minor det***. i.e. $\mathbf{D_x}$, $\mathbf{D_y}$, $\mathbf{D_z}$, etc...
3) Set up for minors-→
   - 1^{st} col [x-values], 2^{nd} col [y-values], etc...
   - D_x → x-col gets "constants"—the rest of the col get coeff. Of var.
   - D_y → y-col gets "constants" "
   - D_z → z-col gets "constants" "
4) To find the actual ***var*** values divide the Minor(s) det. By the Major det.

$X = \frac{Dx}{D}$ $y = \frac{Dy}{D}$ $z = \frac{Dz}{D}$ ➔ this quotient yields the solution to the system:

(x, y) or (x, y, z), etc...

Ex 1) Solve the system Using Cramer's Rule➔ $\begin{cases} 4x - 7y = -3 \\ 2x - 3y = 1 \end{cases}$

Coeff→ **D** = $\begin{vmatrix} 4 & -7 \\ 2 & -3 \end{vmatrix}$ ➔ −12 + 14

D = 2

Sol➔ (8, 5)

$\mathbf{D_x} = \begin{vmatrix} -3 & -7 \\ 1 & -3 \end{vmatrix}$

$\mathbf{D_x = 16}$

X = $\frac{16}{2}$

$\mathbf{D_y} = \begin{vmatrix} 4 & -3 \\ 2 & 1 \end{vmatrix}$

$\mathbf{D_y = 10}$

y = $\frac{10}{2}$

Ex 2) Use Cramer's Rule to solve:

$$\begin{cases} \mathbf{3x - 2y + z = 16} \\ \mathbf{2x + 3y - z = -9} \\ \mathbf{x + 4y + 3z = 2} \end{cases}$$

Area of a △ with verticies→ (x_1, y_1), (x_2, y_2), & (x_3, y_3) use ⊕ to always yield +Area.

Set up the ***determinant*** as follows → **A =** $\pm\frac{1}{2}\begin{vmatrix} x1 & y1 & 1 \\ x2 & y2 & 1 \\ x3 & y3 & 1 \end{vmatrix}$

Example of Using *determinants* to find the area of a triangle given the 3-vertices.

Use ***determinants*** to find the **Area of a Triangle** with vertices (9, 8), (–5, –2), & (1, –6).

A = $\pm\frac{1}{2}\begin{vmatrix} 9 & 8 & 1 \\ -5 & -2 & 1 \\ 1 & -6 & 1 \end{vmatrix}$ [[evaluate by *diagonal method* OR a *calculator*→ det(A)= 116]]

$= \pm\frac{1}{2}(116)$

Show work for ***diagonal method* below

A = 58 square units