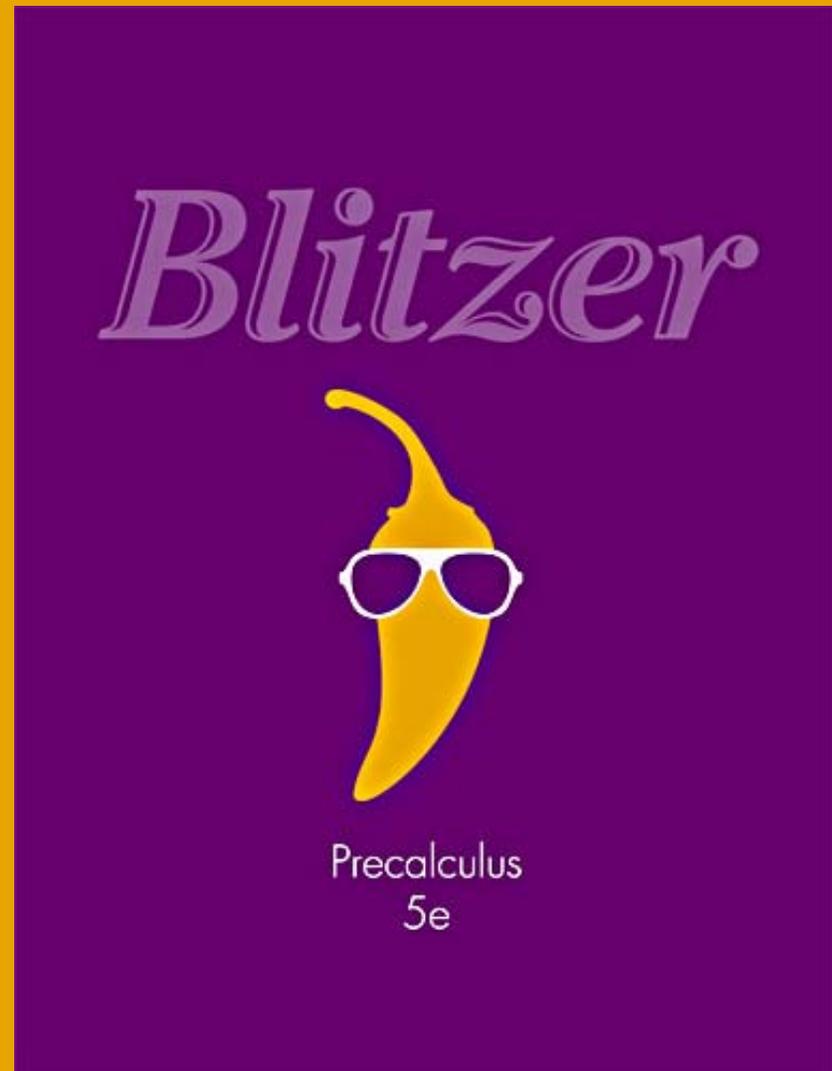


Chapter 1

Functions and Graphs

1.2 Basics of Functions and Their Graphs



Objectives:

- Find the domain and range of a relation.
- Determine whether a relation is a function.
- Determine whether an equation represents a function.
- Evaluate a function.
- Graph functions by plotting points.
- Use the vertical line test to identify functions.
- Obtain information about a function from its graph.
- Identify the domain and range of a function from its graph.
- Identify intercepts from a function's graph.

Definition of a Relation

A **relation** is any set of ordered pairs. The set of all first components of the ordered pairs is called the **domain** of the relation and the set of all second components is called the **range** of the relation.

Example: Finding the Domain and Range of a Relation

Find the domain and range of the relation:

$\{(0, 9.1), (10, 6.7), (20, 10.7), (30, 13.2), (40, 21.2)\}$.

domain: $\{0, 10, 20, 30, 40\}$

range: $\{9.1, 6.7, 10.7, 13.2, 21.2\}$

Definition of a Function

A **function** is a correspondence from a first set, called the **domain**, to a second set, called the **range**, such that each element in the domain corresponds to *exactly one* element in the range.

Example: Determining Whether a Relation is a Function

Determine whether the relation is a function:

$\{(1, 2), (3, 4), (6, 5), (8, 5)\}$.

No two ordered pairs in the given relation have the same first component and different second components. Thus, the relation is a function.

Functions as Equations

If an equation is solved for y and more than one value of y can be obtained for a given x , then the equation does not define y as a function of x .

Example: Determining Whether an Equation Represents a Function

Determine whether the equation defines y as a function of x .

$$x^2 + y^2 = 1$$

$$y^2 = 1 - x^2$$

$$y = \pm\sqrt{1 - x^2}$$

The \pm shows that for certain values of x , there are two values of y . For this reason, the equation does not define y as a function of x .

Function Notation

The special notation $f(x)$, read “ f of x ” or “ f at x ”, represents the value of the function at the number x .

Example: Evaluating a Function

If $f(x) = x^2 - 2x + 7$, evaluate $f(-5)$.

$$f(x) = x^2 - 2x + 7,$$

$$f(-5) = (-5)^2 - 2(-5) + 7 = 25 + 10 + 7 = 42$$

Thus, $f(-5) = 42$.

Graphs of Functions

The **graph of a function** is the graph of its ordered pairs.

Example: Graphing Functions

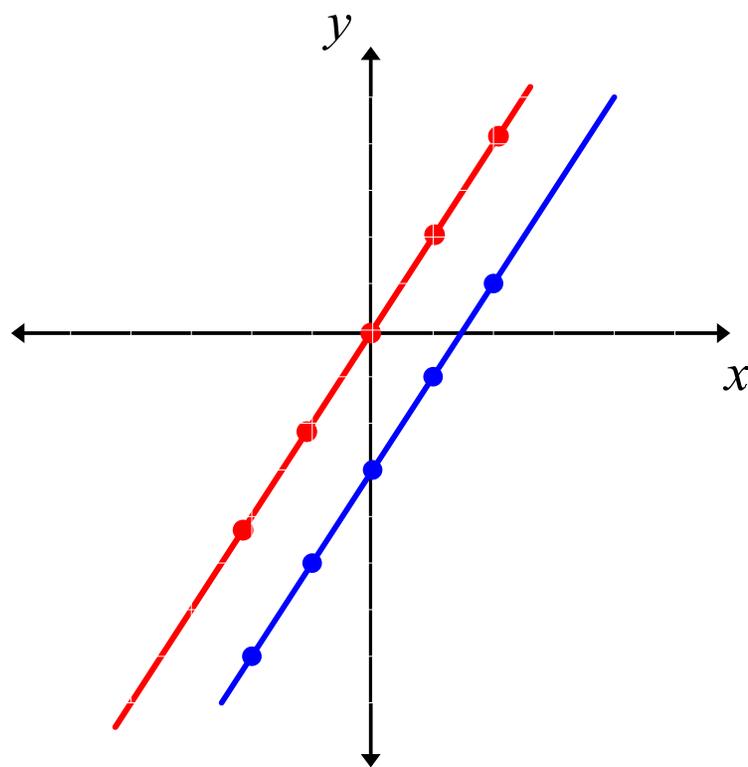
Graph the functions $f(x) = 2x$ and $g(x) = 2x - 3$ in the same rectangular coordinate system. Select integers for x , starting with -2 and ending with 2 .

Example: Graphing Functions (*continued*)

We set up a partial table of coordinates for each function. We then plot the points and connect them.

$$f(x) = 2x$$

x	$y = f(x)$
-2	-4
-1	-2
0	0
1	2
2	4



$$g(x) = 2x - 3$$

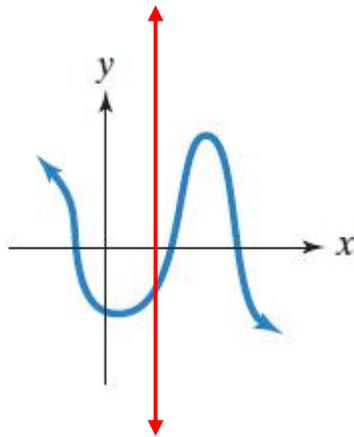
x	$y = f(x)$
-2	-7
-1	-5
0	-3
1	-1
2	1

The Vertical Line Test for Functions

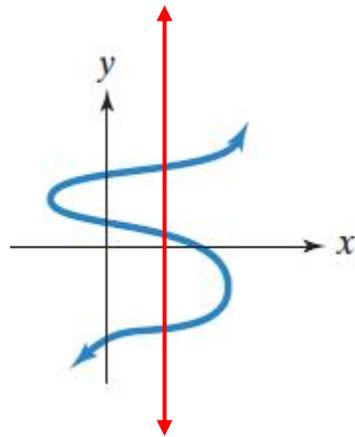
If any vertical line intersects a graph in more than one point, the graph does not define y as a function of x .

Example: Using the Vertical Line Test

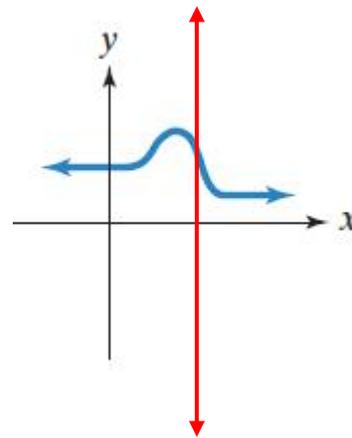
Use the vertical line test to identify graphs in which y is a function of x .



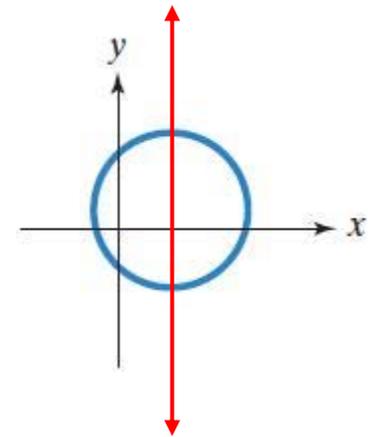
function



not a function



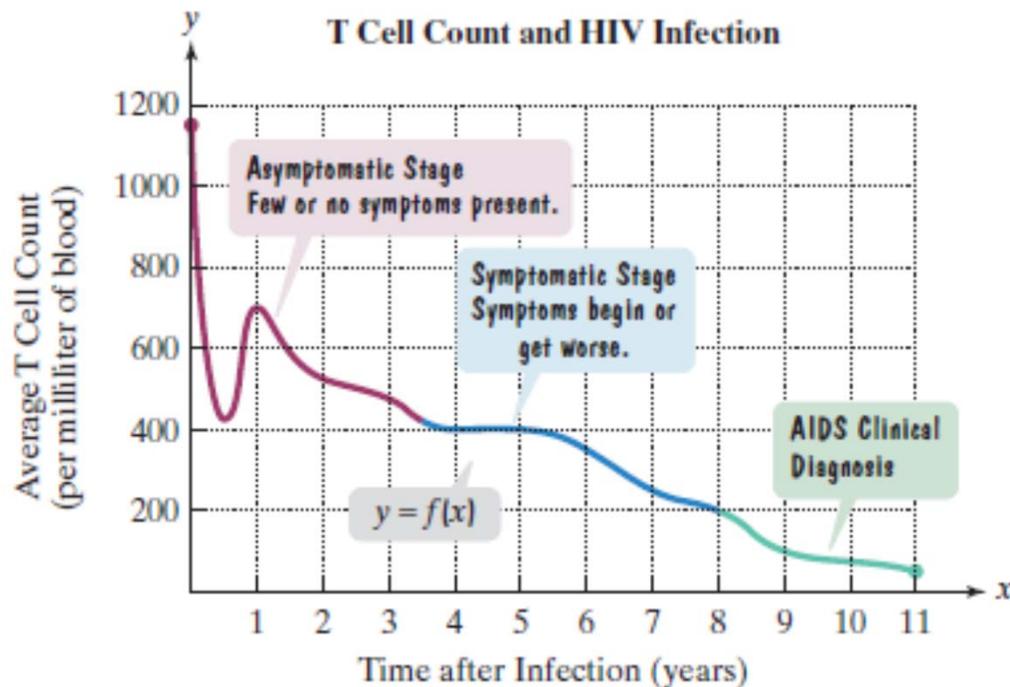
function



not a function

Example: Analyzing the Graph of a Function

Use the graph to find $f(5)$



$$f(5) = 400$$

For what value of x is $f(x) = 100$?

$f(9) = 125$, so $x = 9$.

Identifying Domain and Range from a Function's Graph

To find the domain of a function from its graph, look for all the inputs on the x -axis that correspond to points on the graph.

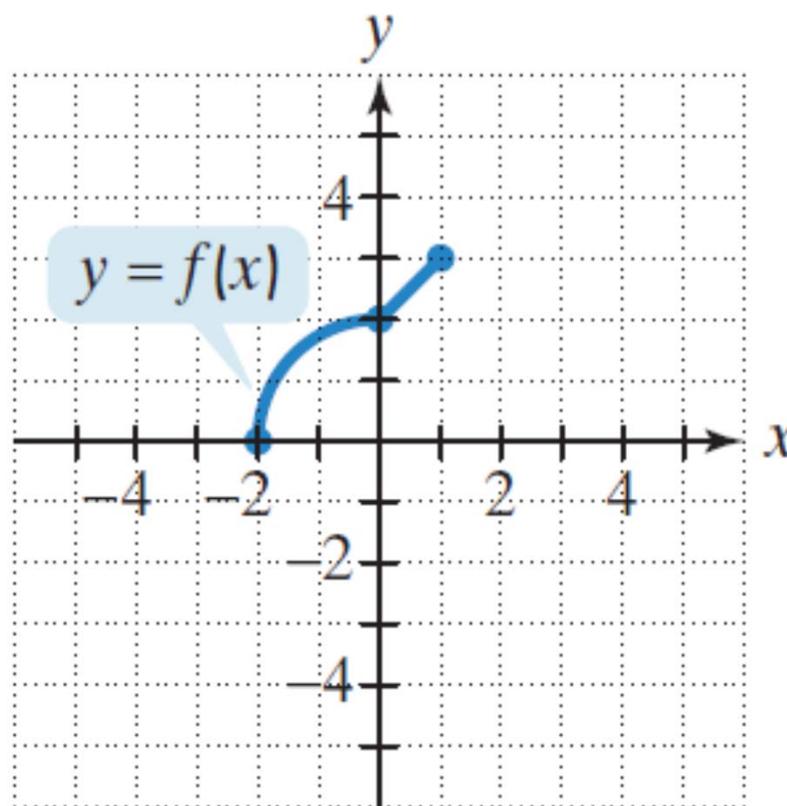
To find the range of a function from its graph, look for all the outputs on the y -axis that correspond to points on the graph.

Example: Identifying the Domain and Range of a Function from Its Graph

Use the graph of the function to identify its domain and its range.

$$\text{Domain } \{x \mid -2 \leq x \leq 1\}$$
$$[-2, 1]$$

$$\text{Range } \{y \mid 0 \leq y \leq 3\}$$
$$[0, 3]$$

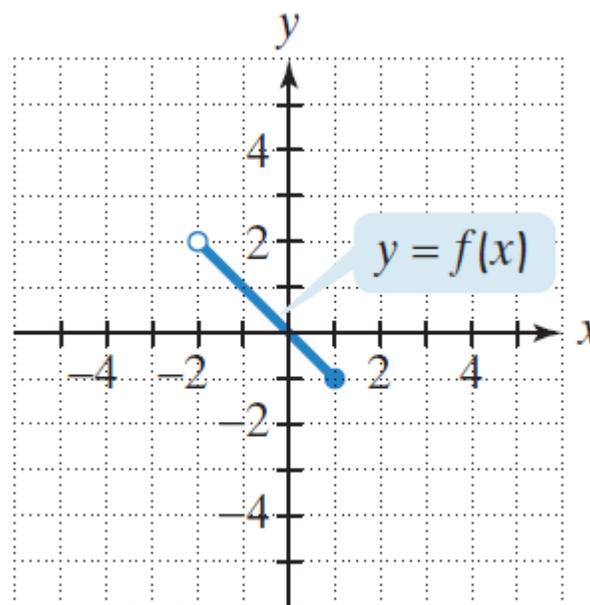


Example: Identifying the Domain and Range of a Function from Its Graph

Use the graph of the function to identify its domain and its range.

$$\text{Domain } \{x \mid -2 < x \leq 1\}$$
$$(-2, 1]$$

$$\text{Range } \{y \mid -1 \leq y < 2\}$$
$$[-1, 2)$$



Identifying Intercepts from a Function's Graph

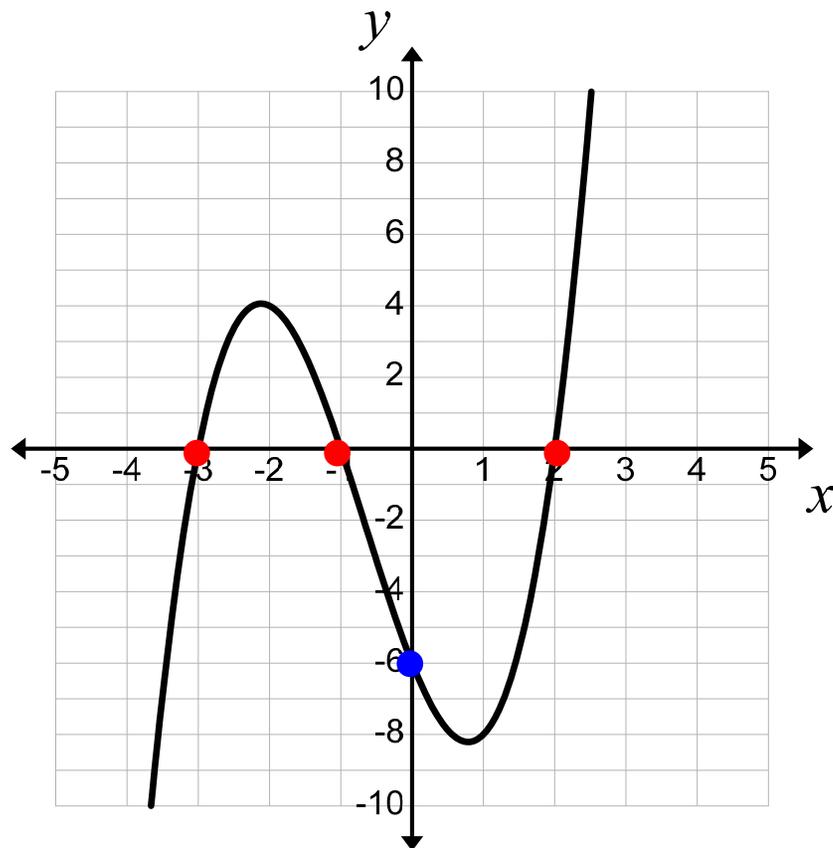
To find the x -intercepts, look for the points at which the graph crosses the x -axis.

To find the y -intercept, look for the point at which the graph crosses the y -axis.

A function can have more than one x -intercept but at most one y -intercept.

Example: Identifying Intercepts from a Function's Graph

Identify the x - and y -intercepts for the graph of $f(x)$.



The x -intercepts are
 $(-3, 0)$
 $(-1, 0)$
and $(2, 0)$.

The y -intercept is $(0, -6)$.