Pre-Calculus - Chapter 3 <u>Sections 3.1-3.4</u>- Notes

Properties of Exponents (Review)

1. $(b^x)(b^y) = b^{x+y}$
2. $(b^x)^y = b^{xy}$, $(abc)^y = a^y b^y c^y$
3. $b^0 = 1$
4. $b^{-x} = 1/(b^x)$
5. $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$
6. $\frac{a^x}{a^y} = a^{x-y}$

Exponential Functions (Section 3.1)

<u>Definition of Exponential Functions</u>

The function f defined by $f(x) = b^x$, where

- $b > 0$
- $b \neq 1$
- the exponent is x (any real number).

<u>Characteristics of Exponential Functions</u>

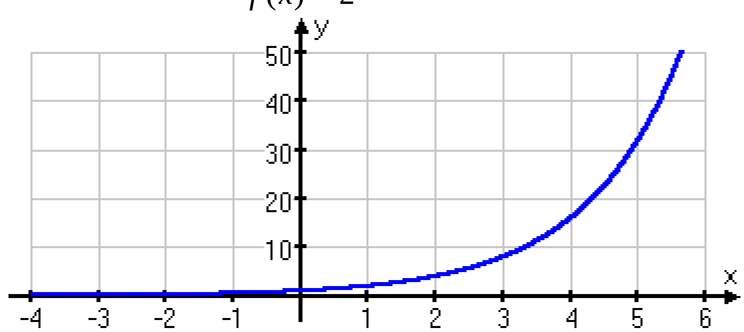
- the y-intercept is (0,1).
- the domain is all real numbers.
- the range is y>0.
- the x- axis is the horizontal asymptote.
- there are no vertical asymptotes.

Again, note that the variable *x* is in the exponent as opposed to the base, and the base *b* is restricted to being a positive number other than 1.

- Also because b>0 and $b \neq 1$, then we can reach the conclusion that either **0<b<1 or b>1.**

Exponential function when b>1:

$f(x) = 2^x$



The values of this function "start" small — very small, so small that they're practically indistinguishable from "$y = 0$", which is the *x*-axis— and then, once they start growing, they grow faster and faster, so fast that they shoot right up through the top of your graph.
This is an **increasing function** and describes **<u>exponential growth (See 3.5)</u>.**
Ex: population growth or money in a savings account).

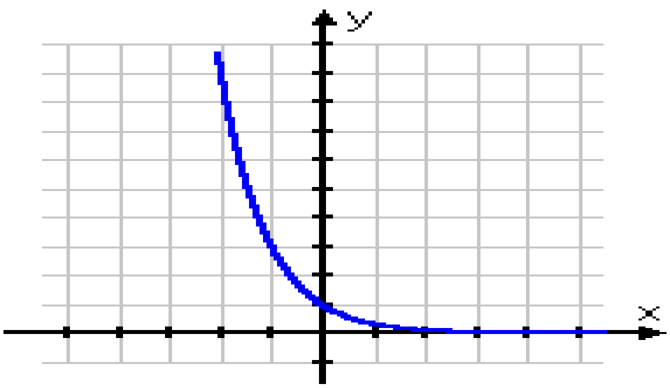
You may have heard of the term "exponential growth".**(<u>Section 3.5</u>)**. Our function f(*x*) above doubled each time we incremented *x*. That is, when *x* was increased by 1, *y* increased to twice what it had been. This is the definition of exponential growth: that there is a consistent fixed period during which the function will double (or triple, or quadruple, etc). So if you hear somebody claiming that the world population is doubling every thirty years, you know he is claiming exponential growth.

Compound Interest: $A = P\left(1+\frac{r}{n}\right)^{nt}$ **Continuous Compouding:** $A = Pe^{rt}$

where A= Final Amount, P= Principal, r=rate, n= number of times interest is paid or compounded each year and t=number of years.

Exponential function when 0<b<1:

$$f(x) = \left(\frac{1}{3}\right)^x$$



The values of this function "start" very large, getting smaller and smaller by half (or a third, or...) at each step, so small that they're practically indistinguishable from "$y = 0$", which is the x-axis .
This is a **decreasing function** and describes **<u>exponential decay (See 3.5).</u>**
Ex: when a quantity loses value exponentially over time).

By looking at an exponential equation or its graph, you will be able to correctly identify which type of change it represents, growth or decay. **<u>(See section 3.5)</u>**

<u>Transformations of Exponential Functions</u>: (See Page 392)

As with other types of graphs, $y = b^x$ represents a parent graph. The same techniques used to transform the graphs of other functions you have studied can be applied to graphs of exponential functions.
Examples:

- $f(x) = 3^{x+c}$ means the parent graph has been shifted c units to the left.
- $f(x) = 3^{x-c}$ means the parent graph has been shifted c units to the right.
- $f(x) = 5^x + c$ means the parent graph has been shifted c units up.
- $f(x) = 5^x - c$ means the parent graph has been shifted c units down.
- $f(x) = 3^{x-a} - b$ means the parent graph has been shifted a units to the right and b units down.
- $f(x) = -b^x$ means the parent graph has been reflected over the x-axis.
- $f(x) = b^{-x}$ means the parent graph has been reflected over the y-axis.

When graphs are translated, the y-intercept and the asymptote move with the graph. Remember: to find the y-intercept of the graph of any function or relation, you must set x=0 and solve for "y".

<u>Section 3.1 Pg. 393</u>

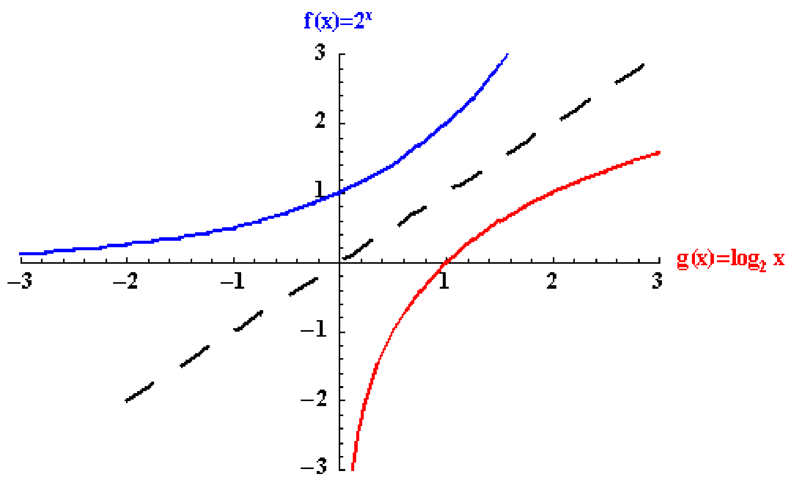
The number ***"e"*** is an irrational number which appears as the base of many exponential functions.

The number ***"e"*** is defined as the value that $\left(1+\frac{1}{n}\right)^n$ approaches as n gets larger and larger.

As $n \to \infty$ the approximate value of e to 4 decimal places is $\boxed{e \approx 2.7183}$

<u>Section 3.2 Pg. 400</u> **Logarithms:**

Logarithmic functions are the **inverse of exponential functions**. For example if (4, 16) is a point on the graph of an exponential function, then (16, 4) would be the corresponding point on the graph of the inverse logarithmic function.



<u>Logarithmic functions:</u>

$x = b^y$ or $\log_b x = y$, where

- b > 0
- b ≠ 1

<u>Characteristics of Logarithmic functions:</u>

- the x-intercept is (1,0).
- the domain is x>0.
- the range is all real numbers.
- the y- axis is the vertical asymptote.
- there are no horizontal asymptotes.

Compare the characteristics of logarithmic functions to the characteristics of exponential functions.

<u>Need to know how to:</u>

- **Evaluate log functions.**
- **Graph log functions.**
- **Find the domain of log functions.**
- **Change from log function to exponential function.**
- **Change from exponential function to log function.**

Common Logarithms and Natural Logarithms.

- The most common bases are the base 10 and the base *e*. Logarithms with a base 10 are called **common logarithms**, and logarithms with a base *e* are **natural logarithms**. The techniques to simplify expressions and solve equations with logarithms are the same, regardless of the base.

  On your calculator, the base 10 logarithm is noted by **log**, and the base *e* logarithm is noted by **ln**. $(\ln = \log_e)$.
- Properties of **common** and **natural** logs. See pages 407 & 409.

SECTION 3.3 Properties of Logarithms.

EXPANDING AND COMPRESSING LOG EXPRESSCIONS

m and ***n*** are positive numbers, ***b*** is a positive number other than 1, and ***p*** is any real number.

Property	**Definition**
Product	$\log_b mn = \log_b m + \log_b n$
Quotient	$\log_b \frac{m}{n} = \log_b m - \log_b n$
Power	$\log_b m^p = p \log_b m$
Equality	If $\log_b m = \log_b n$, then $m = n$

CHANGING THE BASE OF A LOGARITHM

There are an infinite number of bases and only a few buttons on your calculator. You can convert a logarithm with a base that is not 10 or *e* to an equivalent logarithm with base 10 or e.

Let a, b, and x be positive real numbers such that $a \neq 1$ and $b \neq 1$ (remember x must be greater than 0). Then $\log_a x$ can be converted to the base b by the formula

$$\frac{\log_b x}{\log_b a}$$

<u>Example of CHANGE OF BASES</u>: **Find $\log_3 7$ to an accuracy of six decimals.**

Note that the answer will be between 1 and 2 because $3^1 = 3$and $3^2 = 9$, and 7 is between 3 and 9. According to the change of logarithm rule, $\log_3 7$ can be written as $\frac{\log_{10} 7}{\log_{10} 3}$. When the base is 10, we can leave off the 10 in the notation. Therefore $\frac{\log_{10} 7}{\log_{10} 3}$ can be written $\frac{\log 7}{\log 3}$. Use your calculator to find the answer. (Ans: 1.77124).

Let's check the answer. If $3^{1.77124} = 7$, our answer is correct. $3^{1.77124} = 6.999971$. Close enough. Why isn't it 7 exactly? **<u>Note:</u>** The Change of Bases Property works with any base, as long as the bases are the same. For calculator purposes, we can only use base 10 or *e*.

ADD YOUR OWN NOTES FOR 3.4 <u>SOLVING EXP. AND LOG FUNCTIONS</u>
AND 3.5 <u>EXPONENTIAL MODELING: GROWTH AND DECAY.</u>

<u>Miscellaneous Notes:</u>

If unable to solve an equation or simplify an expression "as is", try one of the following:

1) Change from exponential form to "log" or "ln" form.
2) Change from "log" or "ln" form to exponential form.
3) Drop "log" or "ln" from both sides of an equation if there is a single "log" or "ln" on each side and the bases are the same.
4) Insert "log" or "ln" on both sides of an equation.
5) Perform a change of bases.
6) Expand the expression using the properties of logs.
7) Compress the expression (write as a single log) using the properties of logs.
8) Know how your calculator works using "log", "ln", and "e".
9) When solving equations, check the solutions (in case any of them do not work).
10)

$\log 10^0 = 0$	$\ln e^0 = 0$
$\log 10 = 1$	$\ln e = 1$
$\log 10^2 = 2$	$\ln e^2 = 2$
$\log 10^3 = 3$	$\ln e^3 = 3$
and so on	and so on