## Triangles and Trigonometry

## Triangles

1. In the diagram below of triangle $P Q R, \angle R$ and $\angle Q$ are bisected by $\overline{R B}$ and $\overline{Q A}$, respectively. Segments $R B$ and $Q A$ intersect at point $C$. What is the value of $x$ ?

A. 22
B. 33
C. 60
D. 120
2. In $\triangle A B C, m \angle A=x, m \angle B=2 x+2$, and $m \angle C=3 x+4$. What is the value of $x$ ?
A. 29
B. 31
C. 59
D. 61
3. In the diagram below of triangle $H Q P, H P$ is extended through $P$ to $T, m \angle Q P T=6 x+20, m \angle H Q P=x+40$, and $m \angle P H Q=4 x-5$. Find $m \angle Q P T$.

4. What is the value of $x$ in the figure below?

A. 9.3
B. 15.2
C. 17.5
D. 35

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5. Given $\triangle X Y Z$ with line $k$ containing point $Y$, and $k \| \overline{X Z}$.

Which statement will be most likely be used to prove that $m \angle X+m \angle X Y Z+m \angle Z=180^{\circ}$ ?

A. $m \angle 1=m \angle X$
B. $m \angle 1=m \angle 3$
C. $m \angle 2=90^{\circ}$
D. $m \angle 2+m \angle Z=180^{\circ}$
6. Consider $\triangle P Q R$ with $\overline{A B} \| \overline{P Q}$.


Rashad used the given figure to prove that the interior angles of a triangle add to $180^{\circ}$. An incomplete proof is shown below.

|  | Statements | Reasons |
| :--- | :--- | :--- |
| 1. | $\overline{A B} \\| \overline{P Q}$ | Given |
| 2. | $m \angle A R B=180^{\circ}$ | Definition of straight angle |
| 3. | $m \angle 4+m \angle 3+m \angle 5=180^{\circ}$ | $?$ |
| 4. | $?$ | If two parallel lines are cut by a transversal, then the <br> resulting alternate interior angles are congruent |
| 5. | $m \angle 4=m \angle 1$ and $m \angle 5=m \angle 2$ | Definition of congruence |
| 6. | $m \angle 1+m \angle 3+m \angle 2=180^{\circ}$ | $?$ |

Select the option that completes Rashad's proof.
A. Reason 3: Linear Pair Postulate

Statement 4: $m \angle 4=m \angle 2$ and $m \angle 5=m \angle 1$
Reason 3: Transitive Property
B. Reason 3: Angle Addition Postulate

Statement 4: $m \angle 4=m \angle 1$ and $m \angle 5=m \angle 2$
Reason 3: Substitution Property
C. Reason 3: Angle Addition Postulate

Statement 4: $m \angle 4=m \angle 2$ and $m \angle 5=m \angle 1$
Reason 3: Substitution Property
D. Reason 3: Substitution Property

Statement 4: $m \angle 4=m \angle 1$ and $m \angle 5=m \angle 2$
Reason 3: Transitive Property

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7. In the figure shown, $\overline{Q P} \cong \overline{Q R}$.


What is the value of $x$ ?
$x=\square$
8. In the diagram below of $\triangle A B C, \overline{A B} \cong \overline{A C}$. What is the value of $x$ ?

A. $x=20$
B. $x=35$
C. $x=32$
D. $x=40$
9. If $m \angle Y=(4 x+20)^{\circ}$ and $m \angle X=(6 y-2)^{\circ}$. What is the value of $x+y$ ?

A. $x+y=8$
B. $x+y=9$
C. $x+y=17$
D. $x+y=22$
10. In $\triangle P Q R, m \angle P=m \angle R$. If $Q R=x+40, R P=3 x-15$, and $P Q=2 x+30$, what is the length of $\overline{R P}$ ?
A. $R P=10$
B. $R P=15$
C. $R P=50$
D. $R P=45$

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11. The vertex angle of an isosceles triangle measures 15 degrees more than one of its base angles. How many degrees are there in a base angle of the triangle?
A. 50
B. 55
C. 65
D. 70
12. Isosceles triangle $C B A$ with $\overline{C A} \cong \overline{C B}$ is shown below. Martina wants to prove that $\angle A \cong \angle B$. She joined vertex $C$ with point $D$, the midpoint of side $A B$.


Martina wrote the proof below.

|  | Statements | Reasons |
| :---: | :--- | :--- |
| 1. | $\overline{C A} \cong \overline{C B}$ | Given |
| 2. | $\overline{A D} \cong \overline{B D}$ | $?$ |
| 3. | $\overline{C D} \cong \overline{C D}$ | Reflexive Property |
| 4. | $\Delta A C D \cong \triangle B C D$ | $?$ |
| 5. | $\angle A \cong \angle B$ | $?$ |

Which option shows the correct reasons that complete Martina's proof.
A. Reason 2: Property of isosceles triangles.

Reason 4: Side-Side-Side congruence postulate.
Reason 5: Corresponding parts of similar triangles are congruent.
B. Reason 2: $D$ is the midpoint of $\overline{A B}$.

Reason 4: Side-Side-Side congruence postulate.
Reason 5: Corresponding parts of congruent triangles are congruent.
C. Reason 2: Property of isosceles triangles.

Reason 4: Side-Angle-Side congruence postulate.
Reason 5: Corresponding parts of similar triangles are congruent.
D. Reason 2: $D$ is the midpoint of $\overline{A B}$.

Reason 4: Side-Angle-Side congruence postulate.
Reason 5: Corresponding parts of congruent triangles are congruent.
13. In the diagram below of $\triangle D E F, \overline{A B}$ is a midsegment.

If $A D=4, B E=3$, and $A B=5$, what is the perimeter of $\triangle D E F$ ? Enter your answer in the box.


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14. In the figure $\overline{D F}, \bar{B} \bar{D}$, and $\overline{B F}$ are midsegments of $\triangle A C E$. Which of the following statements are true? Select All that apply.


Note: Figure not drawn to scale.
A. $\quad x=42^{\circ}$
B. $x+y=84^{\circ}$
C. $x+y=138^{\circ}$
D. $A B=4$
E. $\quad A C=4$
15. In the figure $\overline{U V}, \overline{V W}$, and $\overline{U V}$ are midsegments of $\triangle L M N$.

If $L U=3(x-5), V W=3+x$, and $m \angle M L N=(y+15)^{\circ}$, which of the following statements are true? Select All that apply.

A. $\quad x=9$
B. $x=11$
C. $y=25$
D. $y=40$
E. $\quad U M=12$
F. $m \angle U W M=55^{\circ}$
16. In $\triangle P Q R$, point $C$ is the centroid, and $P C=27$. What is $C Y$ ?

A. $\quad 13.5$
B. 27
C. 40.5
D. 54
17. In $\triangle A B C$, shown below, $J P=6$. What is $J C$ ?


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18. In $\triangle A B C$, shown below, $D$ is the centroid. If $D G=8 x-4$ and $C G=16 x+12$, find $D C$.

A. 3
B. 20
C. 40
D. 60
19. Which of the following may represent possible side lengths of a triangle?
A. 11.2, 16.1, and 4.7
B. $12.3,5.8$, and 4.7
C. 8.2, 7.3, and 15.8
D. $5.3,14.6$, and 9.4
20. Karla is designing a triangular banner. The lengths of two of the sides are 4.5 feet and 9.5 feet. Select all the possible lengths of the third side.
A. 4.5 ft .
B. 5.5 ft .
C. 9.5 ft .
D. 13.5 ft .
E. $\quad 14.5 \mathrm{ft}$.
21. The lengths of two sides of a triangle are 5 feet and 14 feet. What is the range of possible lengths for the third side?
A. 8 feet $<x<18$ feet
B. 9 feet $<x<18$ feet
C. 9 feet $<x<19$ feet
D. 10 feet $<x<20$ feet
22. The perimeter of $\Delta J K L$ is 200 centimeters.

- $J K=68 \mathrm{~cm}$.
- $K L=62 \mathrm{~cm}$.

Which statement is true about the interior angles of $\Delta J K L$ ?
A. $m \angle L$ is the greatest of the interior angles.
B. $m \angle L$ is the least of the interior angles.
C. $m \angle K$ is the greatest of the interior angles.
D. $m \angle K$ is the least of the interior angles.

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23. In the figure below, $\triangle R S T$ is isosceles, and $\overline{T U} \perp \overline{R S}$.


To prove that base angles of an isosceles triangle are congruent, Madison first proved $\Delta T U R \cong \Delta T U S$. She then concluded $\angle R \cong \angle S$. Which of the following reasons supports this conclusion?
A. Vertical angles are congruent.
B. Side-Side-Angle congruence theorem.
C. Corresponding angles of congruent triangles are congruent.
D. The sum of the measures of the 2 angles that form a linear pair is $180^{\circ}$.

## Trigonometry

24. Based on the figure below, which of the statements are true? Select All that apply.

A. $m \angle A B F=45^{\circ}$
B. $m \angle D C E=30^{\circ}$
C. $B F=26$
D. $C D=26$
E. $C D=39$
F. $D E=13 \sqrt{2}$
25. In the figure shown, point $M$ is the midpoint of $\overline{R S}$.


Which value best represents the length of $\overline{R M}$ ?
A. 41.6
B. 24.0
C. 20.8
D. 12.0

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26. The map below shows the intersections of Main Street, First Avenue, and Canal Street.


What is the distance, in feet, on Main Street between First Avenue and Canal Street? Round your answer to the nearest hundredth.
A. 173.21 ft .
B. 346.41 ft .
C. 424.26 ft .
D. 600 ft .
27. The figure shown is a regular hexagon. If $A B=4 \mathrm{in}$. and $m \angle B=120^{\circ}$, what is the length of diagonal $A C$ ?

A. $4 \sqrt{3}$ in.
B. 8 in .
C. 12 in .
D. $8 \sqrt{3} \mathrm{in}$.
28. In the figure below $\triangle A B C \sim \triangle D E F$. Which of the following statements are true? Select All that apply.


A. $\sin C=\frac{8}{10}$
B. $\tan F=\frac{4}{3}$
C. $\cos D=\frac{6}{8}$
D. $\cos C=\frac{3}{5}$
E. $\tan A=\frac{3}{4}$
29. In right triangle LMN shown below, $m \angle M=90^{\circ}, M N=12$, and $L M=16$. Which of the following statements are true? Select All that apply.
A. $\cos N=\frac{12}{20}$

B. $\cos N=\frac{12}{16}$
C. $\cos N=\frac{16}{20}$
D. $m \angle L=\tan ^{-1}\left(\frac{16}{20}\right)$
E. $m \angle L=\tan ^{-1}\left(\frac{12}{20}\right)$
F. $m \angle L=\tan ^{-1}\left(\frac{12}{16}\right)$

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30. A bird flew from a point on the ground directly to the edge of the roof of the building. The height of the building is 40 feet, and the angle of elevation the bird's flight path made with the ground is $26^{\circ}$. Which expression models the total distance, in feet, the bird flew?
A. $\frac{40}{\cos 26^{\circ}}$
B. $\frac{40}{\sin 26^{\circ}}$
C. $\frac{\cos 26^{\circ}}{40}$
D. $\frac{\sin 26^{\circ}}{40}$
31. You are skiing on a hill and your altitude above the bottom of the hill is 800 feet. The angle of depression is $15^{\circ}$. Find the distance $x$ from you to the base of the hill. Round your answer to the nearest whole foot.

A. 207 ft .
B. 828 ft .
C. 2986 ft .
D. 3091 ft .
32. In right triangle $X Y Z$, the length of hypotenuse $\overline{Y Z}$ is 85 inches and $\tan Z=\frac{3}{4}$. What are the lengths, in inches, of the legs $\overline{X Y}$ and $\overline{X Z}$ ? Enter your answers in the spaces provided.

$$
\begin{aligned}
& X Y=\square \\
& X Z=\square
\end{aligned}
$$

33. The figure represents the side view of a rectangular frame for metal shelves. Two diagonal braces support the frame.


What is the closest to the measure of $x$ ?
A. $7^{\circ}$
B. $14^{\circ}$
C. $28^{\circ}$
D. $76^{\circ}$

2 ft

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34. A fence forms a right triangle with the edges of a campground and a picnic area, as shown below, with dimensions in feet.


Part A: Which of the following equations can be used to determine the value of $x$ ? Select All that apply.
A. $\sin x=\frac{14}{48}$
B. $\sin x=\frac{14}{50}$
C. $\tan x=\frac{48}{14}$
D. $\tan x=\frac{48}{50}$
E. $\cos x=\frac{48}{50}$

Part B: What is the value of $x$ ? Round your answer to the nearest tenth.

$$
x=\square
$$

35. Jeremy likes to ride his bike to his friend Khaleel's house. If he takes the road, he rides 3.6 miles east and then 1.5 miles north. There is also a path that goes through the woods directly from Jeremy's house to Khaleel's house.


Part A. To the nearest degree, what is the angle shown between the road and the path? Use trigonometric functions to calculate the angle.


Part B. To the nearest tenth of a mile, how much farther is it to go by the road than to go by the path? Show your work.
36. As shown in the diagram below, a symmetrical roof frame rises 4 feet above a house and has a width of 24 feet.


Determine and state, to the nearest degree, the angle of elevation of the roof frame. Enter your answer in the box.


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37. Each wire that supports a 300 -foot radio tower forms a $51^{\circ}$ angle with the ground.


Find the distance from the base of the tower to the point on the ground where the wires are secured. Round your answer to the nearest tenth of a foot.
A. 233.1 ft .
B. 242.9 ft .
C. 370.5 ft .
D. 386.0 ft .
38. Marisol's backyard is shaped like an isosceles trapezoid.


110 ft.

Find the distance, $h$, between the two parallel sides of Marisol's backyard. Round your answer to the nearest foot.
A. 10 ft .
B. 18 ft .
C. 41 ft .
D. 82 ft .

## Inscribed and Circumscribed Circles of a Triangle.

39. In order to find the circumcenter of a triangle, for which lines must you find the intersection?
A. the medians of the triangle
B. the angle bisectors of the triangle
C. the perpendicular bisectors of the sides of the triangle
D. the altitudes of the triangle
40. Olivia is constructing the circumscribed circle of a triangle as shown in the diagram. What should be her next step in the process?

A. Construct the angle bisector of $\angle A$.
B. Construct the perpendicular bisector of $\overline{B C}$.
C. Set the compass width to $A B$, then draw a circle with center point $A$.
D. Set the compass width to $B C$, then draw a circle with center point $C$.

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41. The first few steps to construct a circle inscribed in triangle $P R Q$ are shown.


Layla and her friend Devon make the following claims about how they can identify the nest step in the construction.

- Layla: Use the fact that the radius drawn to the point of tangency is perpendicular to the tangent line of a circle.
- Devon: Use the fact that the radius drawn perpendicular to a chord bisects the cord.

Complete the sentence to create a statement about who is correct.
[ $(A)$ Layla (B)Devon] is correct because this fact can be used to determine the length of the [©A radius (B)chord] of the circle by constructing a perpendicular line from point $[A C B C B]$ to any sides of the triangle.
42. Complete the sentences about the diagram shown.


This part of the construction of [ (A) an angle bisector (B) a perpendicular bisector] is needed to construct the [ (A) circumscribed circle of the triangle (B) inscribed circle of the triangle].

