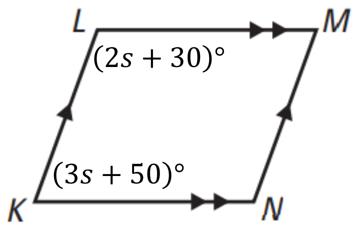
Quadrilaterals and Logic

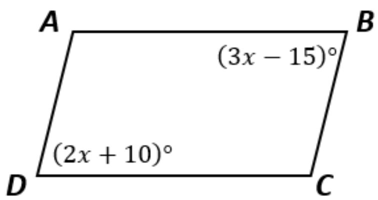
Quadrilaterals

1. Given parallelogram $KLMN$, what is the value of s?



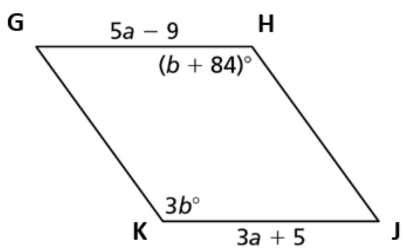
A. 5
B. 20
C. 40
D. 70

2. Given parallelogram $ABCD$ below, find the value of x.



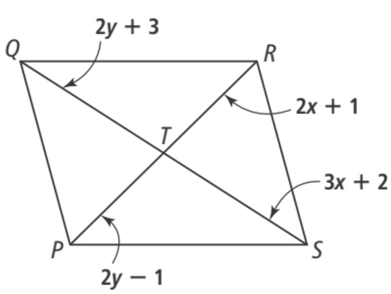
A. 5
B. 25
C. 35
D. 41

3. Given parallelogram $GHJK$, find the values of a and b.



A. $a = 2$ and $b = 24$
B. $a = 2$ and $b = 42$
C. $a = 7$ and $b = 24$
D. $a = 7$ and $b = 42$

4. Given parallelogram $PQRS$ below, select all the true statements.



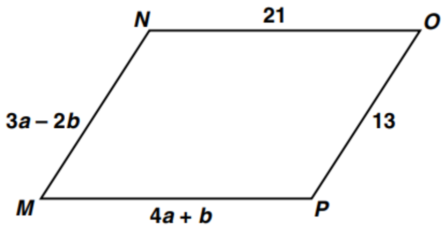
A. $x = 1$
B. $y = 0.5$
C. $QS = 9$
D. $PR = 14$
E. $QS = 22$

5. In parallelogram $ABCD$, $AC = DB$, $m\angle ABC = 2xy$, and $m\angle BCD = 9x + 9$. Find the value of y.

A. 9
B. 5
C. 10
D. 18

Quadrilaterals and Logic

6. What values of a and b make quadrilateral $MNOP$ a parallelogram?



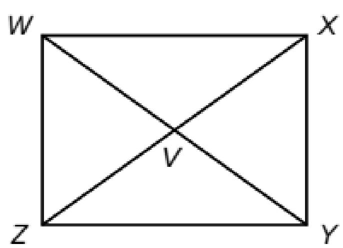
A. $a = 1$ and $b = 5$

B. $a = 5$ and $b = 1$

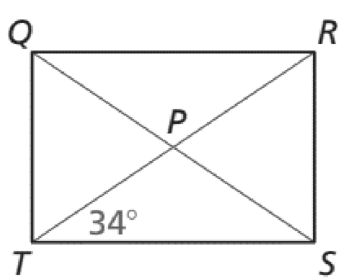
C. $a = \frac{11}{7}$ and $b = \frac{34}{7}$

D. $a = \frac{34}{7}$ and $b = \frac{11}{7}$

7. Given rectangle $WXYZ$ below with $WV = 18$, find XZ. Enter your answer in the box.



8. Given rectangle $QRST$ with $m\angle PTS = 34°$ and $QS = 10$, select All the true statements.



A. $m\angle QTR = 56°$

B. $m\angle QRT = 34°$

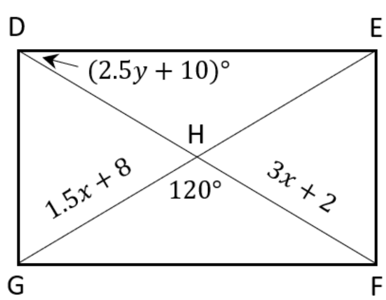
C. $m\angle SRT = 34°$

D. $QP = 5$

E. $RT = 10$

F. $RP = 5$

9. Given rectangle $DEFG$ below, select all the true statements.



A. $x + y = 12$

B. $DH = 14$

C. $m\angle GDH = 30°$

D. $m\angle FEH = 60°$

E. $m\angle FHE = 60°$

Quadrilaterals and Logic

10. The diagonals of square $ABCD$ intersect at point E. If $BE = 2x + 1$ and $AC = 3(6x - 4)$, what is the length of BD?

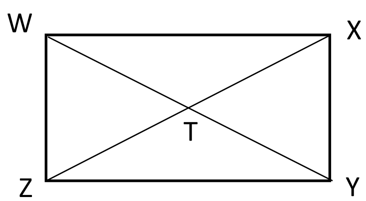
   A. 1 unit

   B. 3 units

   C. 6 units

   D. 9 units

11. In rectangle $WXYZ$, $WY = 4x - 15$ and $XZ = 3x + 8$. Select All the true statements about $WXYZ$.



   A. $x = 7$

   B. $WY = 2XT$

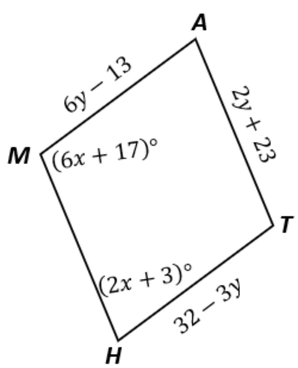
   C. $ZT = \frac{1}{2}WY$

   D. $ZT = TY$

   E. $WY = 77$

   F. $XZ = 77$

12. Given parallelogram $MATH$ below, select All the true statements.



   A. $x = 5$

   B. $x = 20$

   C. $y = 5$

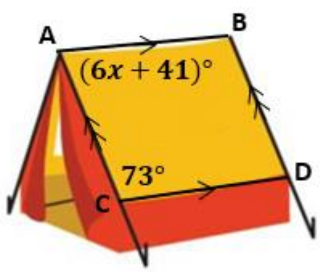
   D. $y = 9$

   E. $m\angle A = 43°$

   F. $m\angle T = 43°$

   G. $MA = 41$

13. Paulina bought a camping tent with a parallelogram-shaped flysheet as drawn below. If $m\angle ACD = 73°$, what is the value of x? Enter your answer in the box.



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14. One angle of an isosceles trapezoid has a measure of 53°. What are the measures of the other three angles of the trapezoid?

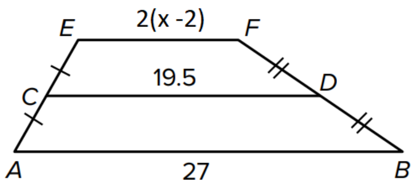
    A. 37°, 127°, 127°

    B. 53°, 127°, 127°

    C. 37°, 135°, 135°

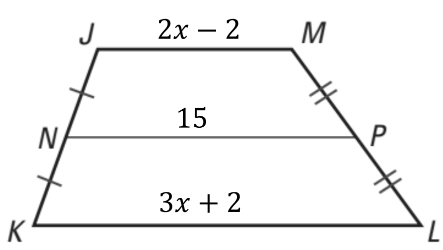
    D. 53°, 82°, 82°

15. Segment CD is the midsegment of trapezoid $AEFB$. What is the value of x?



    A. 7.5
    B. 8
    C. 12
    D. 23.5

16. In the trapezoid below, $NP = 15$. Select All the true statements.



    A. $\overline{NP}$ is the midsegment of trapezoid $JMLK$.

    B. $\overline{NP} \parallel \overline{JM}$

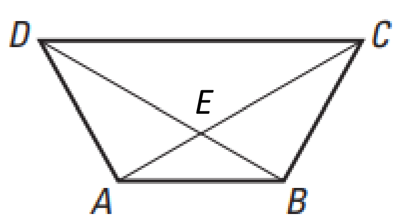
    C. $\overline{NP} \parallel \overline{KL}$

    D. $x = 3$

    E. $JM = 10$

    F. $KL = 11$

17. Given isosceles trapezoid $ABCD$ with diagonals intersecting at point E, select all the true statements.



    A. $\overline{AB} \parallel \overline{DC}$

    B. $\overline{AD} \cong \overline{BC}$

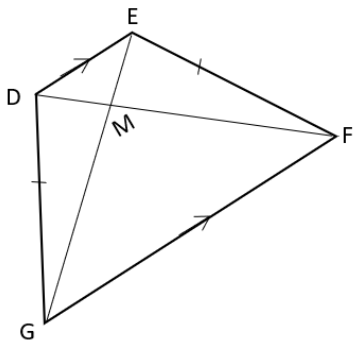
    C. $\overline{DB} \cong \overline{CA}$

    D. $DE = \frac{1}{2}DB$

    E. $AE = DB - DE$

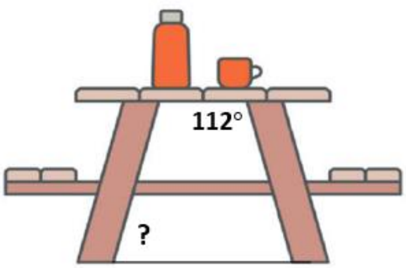
Quadrilaterals and Logic

18. Given isosceles trapezoid $DEFG$ with $DF = 7x + 24$, $EG = 5x + 38$ and $DM = 18$, select All the true statements.

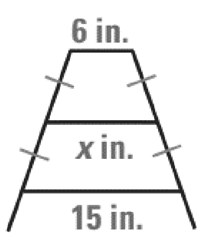


A. $EM = 18$
B. $MF = 36$
C. $DF = 54$
D. $MG = 55$
E. $EG = 73$

19. A manufacturer designed a picnic table as shown below. If the tabletop is parallel to the ground, what is the measure of the angle the table leg makes with the ground? Enter your answer in the box.



20. You want to build a plant stand with three equally spaced circular shelves. You want the top shelf to have a diameter of 6 inches and the bottom shelf to have a diameter of 15 inches. The diagram below shows a vertical cross section of the plant stand. What is the diameter of the middle shelf? Enter your answer in the box.



21. Quadrilateral $BEST$ has diagonals that intersect at point D. Which statement would not be sufficient to prove quadrilateral $BEST$ is a parallelogram?

A. $\overline{BD} \cong \overline{SD}$ and $\overline{ED} \cong \overline{TD}$
B. $\overline{BE} \cong \overline{ST}$ and $\overline{ES} \cong \overline{TB}$
C. $\overline{ES} \cong \overline{TB}$ and $\overline{BE} \parallel \overline{TS}$
D. $\overline{ES} \parallel \overline{BT}$ and $\overline{BE} \parallel \overline{TS}$

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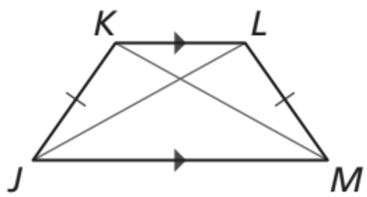
22. The figure shows isosceles trapezoid $JKLM$ with $\overline{KL} \parallel \overline{JM}$ and $\overline{JK} \cong \overline{LM}$. Prove $\overline{JL} \cong \overline{KM}$



	Statements	Reasons
1.	$JKLM$ is an isosceles trapezoid. $\overline{KL} \parallel \overline{JM}$ and $\overline{JK} \cong \overline{LM}$	Given
2.	$\angle JKL \cong \angle MLK$	
3.	$\overline{KL} \cong \overline{KL}$	Reflexive Property of Congruence
4.	$\Delta JKL \cong \Delta MLK$	
5.	$\overline{JL} \cong \overline{KM}$	Corresponding parts of congruent triangles are congruent.

What are the appropriate reasons for the statements in step 2 and step 4?

A. Step 2: Same side Interior Angles Theorem.
Step 4: SSS Congruence Theorem.

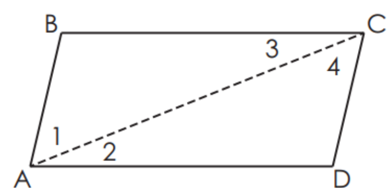
B. Step 2: Same side Interior Angles Theorem.
Step 4: SAS Congruence Theorem.

C. Step 2: Isosceles Trapezoid Base Angles Theorem.
Step 4: SSS Congruence Theorem.

D. Step 2: Isosceles Trapezoid Base Angles Theorem.
Step 4: SAS Congruence Theorem.

23. The proof shows that opposite angles of a parallelogram are congruent.

Given: $ABCD$ is a parallelogram with diagonal $\overline{AC}$.
Prove $\angle BAD \cong \angle DCB$



	Statements	Reasons
1.	$ABCD$ is a parallelogram with diagonal $\overline{AC}$	Given
2.	$\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \parallel \overline{BC}$	Definition of parallelogram
3.	?	Alternate interior angles are congruent.
4.		Measures of congruent angles are equal.
5.	$m\angle 1 + m\angle 2 = m\angle 4 + m\angle 2$	Addition property of equality
6.	$m\angle 1 + m\angle 2 = m\angle 4 + m\angle 3$	?
7.	$m\angle 1 + m\angle 2 = m\angle BAD$ $m\angle 4 + m\angle 3 = m\angle DCB$	
8.	$m\angle BAD = m\angle DCB$	Substitution
9.	$\angle BAD \cong \angle DCB$	Angles are congruent when their measures are equal.

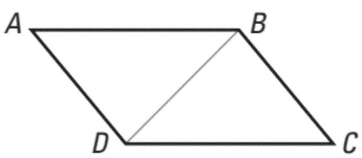
Quadrilaterals and Logic

What are the missing statement and reason in this partial proof?

A. Statement 3: $\angle 2 \cong \angle 4$ and $\angle 1 \cong \angle 3$
   Reason 6: Angle-Side-Angle
B. Statement 3: $\angle 2 \cong \angle 3$ and $\angle 1 \cong \angle 4$
   Reason 6: Substitution
C. Statement 3: $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$
   Reason 6: Angle addition postulate
D. Statement 3: $\angle 1 \cong \angle 4$ and $\angle 2 \cong \angle 3$
   Reason 6: Alternate interior angles are congruent.

24. Maschenka has written an incomplete proof for the congruency of the opposite sides of a parallelogram. Her work is shown below.

Given: $ABCD$ is a parallelogram.
Prove: $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{CB}$



	Statements	Reasons
1.	$ABCD$ is a parallelogram.	Given
2.	$\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \parallel \overline{CB}$	Definition of parallelogram
3.	$\angle ABD \cong \angle CDB$ $\angle ADB \cong \angle CBD$	**?**
4.	$\overline{DB} \cong \overline{DB}$	**?**
5.	**?**	ASA Congruence Postulate
6.	$\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{CB}$	C.P.C.T.C

Which of these could complete the proof?

A. Reason 3: Alternate Interior Angles Theorem
   Reason 4: Reflexive Property of Congruence
   Statement 5: $\Delta ADB \cong \Delta CBD$

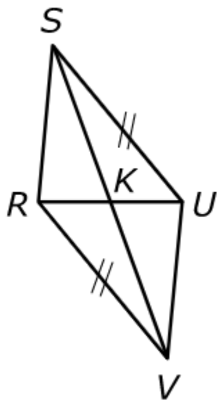
B. Reason 3: Corresponding Angles Theorem
   Reason 4: Transitive Property of Congruence
   Statement 5: $\Delta ADB \cong \Delta CBD$

C. Reason 3: Alternate Interior Angles Theorem
   Reason 4: Reflexive Property of Congruence
   Statement 5: $\Delta ADB \cong \Delta BCD$

D. Reason 3: Corresponding Angles Theorem
   Reason 4: Transitive Property of Congruence
   Statement 5: $\Delta ADB \cong \Delta BCD$

Quadrilaterals and Logic

25. The figure shows quadrilateral $RSUV$ with $\overline{RV} \parallel \overline{SU}$ and congruent segments as marked. Prove that $\overline{RU}$ bisects $\overline{SV}$.



	Statements	Reasons
1.	Quadrilateral $RSUV$ with $\overline{RV} \parallel \overline{SU}$ and $\overline{RV} \cong \overline{SU}$	Given
2.	$\angle KSU \cong \angle KVR$	
3.	$\angle SUK \cong \angle KRV$	
4.	$\Delta SUK \cong \Delta VRK$	
5.	$\overline{SK} \cong \overline{VK}$	
6.	$\overline{RU}$ bisects $\overline{SV}$	Definition of bisect

Part A: What is the appropriate reason for the statement in step 2?

A. Alternate exterior angles formed by parallel lines cut by a transversal are congruent.
B. Alternate interior angles formed by parallel lines cut by a transversal are congruent.
C. Corresponding angles formed by parallel lines cut by a transversal are congruent.
D. Vertical angles are congruent.

Part B: What is the appropriate reason for the statement in step 3?

A. Alternate exterior angles formed by parallel lines cut by a transversal are congruent.
B. Alternate interior angles formed by parallel lines cut by a transversal are congruent.
C. Corresponding angles formed by parallel lines cut by a transversal are congruent.
D. Vertical angles are congruent.

Part C: What is the appropriate reason for the statement in step 4?

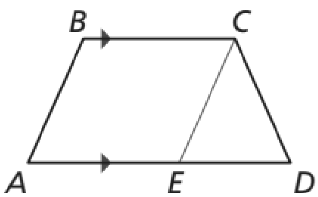
A. Angle Angle Angle
B. Angle Side Angle
C. Angle Side Side
D. Side Side Side

Part D: What is the appropriate reason for the statement in step 5?

A. Corresponding sides of similar triangles are congruent.
B. Corresponding angles of similar triangles are congruent.
C. Corresponding sides of congruent triangles are congruent.
D. Corresponding angles of congruent triangles are congruent.

Quadrilaterals and Logic

26. Given isosceles trapezoid $ABCD$ with $\overline{BC} \parallel \overline{AD}$. Prove $\angle A \cong \angle D$ and $\angle B \cong \angle BCD$.



Given isosceles trapezoid $ABCD$ with $\overline{BC} \parallel \overline{AD}$, construct $\overline{CE}$ parallel to $\overline{AB}$. Then, $ABCE$ is a parallelogram by definition, so $\overline{AB} \cong \overline{EC}$. Because $\overline{AB} \cong \overline{CD}$ by the definition of an Isosceles trapezoid, $\overline{CE} \cong \overline{CD}$ by the _________①___________. So, $\angle CED \cong \angle D$ by the Base Angles Theorem and $\angle A \cong \angle CED$ by the _________②___________. So, $\angle A \cong \angle D$ by the _________③___________. Next, by the Consecutive Interior Angles Theorem, $\angle B$ and $\angle A$ are supplementary and so are $\angle BCD$ and $\angle D$. So, $\angle B \cong \angle BCD$ by the Congruent Supplements Theorem.

What are the appropriate reasons that complete the proof?

A. ① Symmetric Property of Congruence
② Alternate Interior Angles Theorem
③ Transitive Property of Congruence

B. ① Transitive Property of Congruence
② Corresponding Angles Theorem
③ Reflexive Property of Congruence

C. ① Reflexive Property of Congruence
② Alternate Interior Angles Theorem
③ Reflexive Property of Congruence

D. ① Transitive Property of Congruence
② Corresponding Angles Theorem
③ Transitive Property of Congruence

Logic

27. Given the biconditional: *Two lines are perpendicular if and only if they intersect at right angles*.
Write the conditional statement that could be written from the biconditional. What is the converse of that conditional statement?

A. Conditional statement: If two lines intersect at right angles, then they are perpendicular.
Converse: If two lines are not perpendicular, then they do not intersect at right angles.

B. Conditional statement: If two lines are perpendicular, then they intersect at right angles.
Converse: If two lines do not intersect at right angles, then they are not perpendicular.

C. Conditional statement: If two lines intersect at right angles, then they are perpendicular.
Converse: If two lines are perpendicular, then they intersect at right angles.

D. Conditional statement: If two lines are perpendicular, then they intersect at right angles.
Converse: If two lines intersect at right angles, then they are perpendicular.

Quadrilaterals and Logic

28. Rewrite the definition of congruent segments as a single biconditional statement.
Definition: *If two line segments have the same length, then they are congruent segments.*

   A. If two line segments are congruent, then they have the same length.
   B. If two line segments are not congruent, then they don't have the same length.
   C. Two line segments have the same length if and only if they are congruent segments.
   D. Two line segments do not have the same length if and only if they are not congruent segments

29. Rewrite the following definition as a biconditional statement.
Definition: *The midpoint of a segment is the point that divides the segment into two congruent segments.*

   A. If a point is not the midpoint of a segment, then the point doesn't divide the segment into two congruent segments.
   B. A point is the midpoint of a segment if and only if the point divides the segment into two congruent segments.
   C. If a point divides the segment into two congruent segments, then the point is the midpoint.
   D. A point divides the segment into two congruent segments if and only if the point is the midpoint.

30. What is the inverse of the statement, "If a parallelogram has a right angle, then the parallelogram is a rectangle"?

   A. If a parallelogram is a rectangle, then the parallelogram has a right angle.
   B. If a parallelogram is not a rectangle, then the parallelogram does not have right angle.
   C. If a parallelogram does not have a right angle, then the parallelogram is not a rectangle.
   D. If a parallelogram has a right angle, then the parallelogram is not a rectangle.

31. What is the converse of the statement, "If two angles are congruent, then they have the same measure"?

   A. If two angles are not congruent, then they have the same measure.
   B. If two angles are not congruent, then they don't have the same measure.
   C. If two angles have the same measure, then they are congruent.
   D. If two angles don't have the same measure, then they are not congruent.

32. What is the contrapositive of the statement, "If a quadrilateral is a rectangle, then it has two pairs of parallel sides"?

   A. If a quadrilateral is not a rectangle, then it has two pairs of parallel sides.
   B. If a quadrilateral is not a rectangle, then it does not have two pairs of parallel sides.
   C. If a quadrilateral has two pairs of parallel sides, then it is a rectangle.
   D. If a quadrilateral does not have two pairs of parallel sides, then it is not a rectangle.

Quadrilaterals and Logic

33. Given the statement "If two triangles are not similar, their corresponding angles are not congruent", write the inverse, converse, and contrapositive statements.

    A. Inverse: If two triangles are similar, their corresponding angles are congruent.
    Converse: If the corresponding angles are congruent, then the two triangles are similar.
    Contrapositive: If the corresponding angles are not congruent, then the two triangles are not similar.

    B. Inverse: If the corresponding angles are congruent, then the two triangles are similar.
    Converse: If two triangles are similar, their corresponding angles are congruent.
    Contrapositive: If the corresponding angles are not congruent, then the two triangles are not similar.

    C. Inverse: If the corresponding angles are not congruent, then the two triangles are not similar.
    Converse: If two triangles are similar, their corresponding angles are congruent.
    Contrapositive: If the corresponding angles are congruent, then the two triangles are similar.

    D. Inverse: If two triangles are similar, their corresponding angles are congruent.
    Converse: If the corresponding angles are not congruent, then the two triangles are not similar.
    Contrapositive: If the corresponding angles are congruent, then the two triangles are similar.

34. Which figure can serve as a counterexample to the conjecture below?
    "If one pair of opposite sides of a quadrilateral is parallel, then the quadrilateral is a parallelogram."

    A. Rectangle

    B. Rhombus

    C. Square

    D. Trapezoid

35. Which of the options best describes a counterexample to the assertion below?
    "Two lines in a plane always intersect in exactly one point."

    A. coplanar lines

    B. parallel lines

    C. perpendicular lines

    D. intersecting lines

36. Which situation would provide a counterexample to this statement?
    "Alternate interior angles are never supplementary."

    A. A line that is parallel to two parallel lines.

    B. A transversal that forms $45°$ angle with two parallel lines.

    C. A transversal that is perpendicular two parallel lines.

    D. A line that has a slope that is the reciprocal of the slopes of two parallel lines.

Quadrilaterals and Logic

37. Which of the options best describes a counterexample to the assertion below?
"If one pair of opposite sides of a quadrilateral is congruent, then the quadrilateral is a parallelogram."

    A. Isosceles trapezoid
    B. Rectangle
    C. Rhombus
    D. Square

38. Which of the options best describes a counterexample to the statement below?
"The circumcenter of a triangle is always inside the triangle."

    A. The circumcenter is a point of concurrency.
    B. The circumcenter is the point where the perpendicular bisectors of a triangle intersect.
    C. The circumcenter of an obtuse triangle is outside the triangle.
    D. The circumcenter of an equilateral triangle is inside the triangle.

39. Identify a counterexample to the following definition:
If figure is a polygon, then it has four sides.

    A. Rectangle
    B. Rhombus
    C. Square
    D. Triangle

40. Mercedes makes the following conjecture:
If a polygon has sides that are all congruent, then the polygon is a square.

    Which of the following is a counterexample that proves this conjecture to be false?

    A. An isosceles trapezoid.
    B. An isosceles triangle.
    C. A scalene triangle.
    D. An equilateral triangle.