NAME:___________________________ Per:_________

Proving Conditions for Parallelograms (9.2)

As stated in our theorems, parallelograms (quadrilaterals with both pairs of opposite sides parallel) have the following properties:

- The opposite sides are congruent (both pairs).
- The opposite angles are congruent (both pairs).
- The consecutive angles are supplementary (all pairs).
- The diagonals bisect each other.

Conversely, if a quadrilateral has any of the above conditions, we can prove it is a parallelogram. That is, we can prove any of the above conditions will result in a quadrilateral with both pairs of opposite sides parallel.

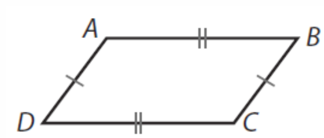
In addition, we have an extra condition that results in a quadrilateral being a parallelogram.

- One pair of sides is both parallel AND congruent.

Condition #1 – The opposite sides are congruent.

Given: $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{CB}$

Prove: $ABCD$ is a parallelogram.

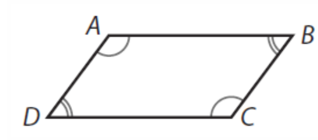


Two column proof (fill in the missing items):

Statements	Reasons
1. Draw $\overline{DB}$.	1. Through any two points, there is exactly one line.
2. $\overline{DB} \cong \overline{DB}$	2.
3. $\overline{AB} \cong \overline{CD}$; $\overline{AD} \cong \overline{CB}$	3.
4. $\triangle ABD \cong \triangle CDB$	4.
5. $\angle ABD \cong \angle CDB$; $\angle ADB \cong \angle CBD$	5.
6. $\overline{AB} \parallel \overline{DC}$; $\overline{AD} \parallel \overline{BC}$	6.
7. $ABCD$ is a parallelogram.	7.

<u>Condition #2 – The opposite angles are congruent.</u>

Given: $\angle A \cong \angle C$ and $\angle B \cong \angle D$ Prove: $ABCD$ is a parallelogram.



Paragraph proof (fill in the missing items):

$m\angle A + m\angle B + m\angle C + m\angle D = 360°$ by ________________________________.

From the given information, $m\angle A = m\angle$ $\square$ and $m\angle B = m\angle$ $\square$. By substitution,

$m\angle A + m\angle D + m\angle A + m\angle D = 360°$ or $2m\angle$ $\square$ $+ 2m\angle$ $\square$ $= 360°$. Dividing

both sides by 2 gives __________________________. Therefore, $\angle A$ and $\angle D$ are

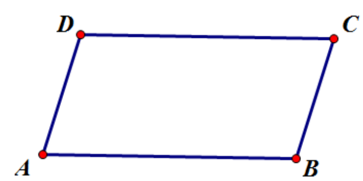
supplementary and so $\overline{AB} \parallel \overline{DC}$ by the ____

A similar argument shows that $\overline{AD} \parallel \overline{BC}$, so $ABCD$ is a parallelogram

by ________________________________.

<u>Condition #3 – Consecutive angles are supplementary.</u>

Given: $\angle A$ and $\angle B$ are supplementary, $\angle B$ and $\angle C$ are supplementary, $\angle C$ and $\angle D$ are supplementary, $\angle D$ and $\angle A$ are supplementary



Prove: ABCD is a parallelogram

Flowchart proof (fill in the missing items):

$\angle A$ and $\angle B$ are supplementary
$\angle D$ and $\angle C$ are supplementary → ____________ → ____________

<u>Given</u>

<u>Converse of the Same Side Interior Angles Postulate</u>

$\angle B$ and $\angle C$ are supplementary
$\angle A$ and $\angle D$ are supplementary → ____________ → ____________

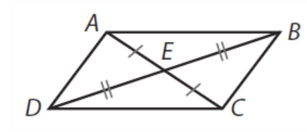
<u>Definition of parallelogram</u>

______________________ ______________________

<u>Condition #4 – The diagonals bisect each other.</u>

Given: $\overline{AE} \cong \overline{CE}$ and $\overline{DE} \cong \overline{BE}$

Prove: $ABCD$ is a parallelogram.



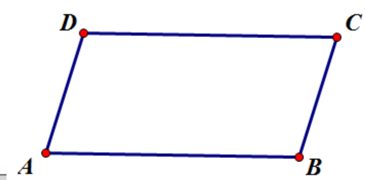
Two column proof (fill in the missing items):

Statements	Reasons
1. $\overline{AE} \cong \overline{CE}$ and $\overline{DE} \cong \overline{BE}$	1.
2. $\angle AEB \cong \angle CED$; $\angle AED \cong \angle CEB$	2.
3. $\Delta AEB \cong \Delta CED$; $\Delta AED \cong \Delta CEB$	3.
4. $\angle ABE \cong \angle CDE$; $\angle EAD \cong \angle ECB$	4.
5. $\overline{AB} \parallel \overline{CD}$; $\overline{AD} \parallel \overline{BC}$	5.
6. ABCD is a parallelogram	6.

<u>Condition #5 – One pair of sides is both parallel AND congruent. **</u>**

Given: $\overline{AB} \cong \overline{CD}$ and $\overline{AB} \parallel \overline{CD}$

Prove: $ABCD$ is a parallelogram. (*Hint*: Draw $\overline{DB}$.)



Two column proof (on your own):

Statements	Reasons
1.	1.