2.6 Proving Geometric Relationships

In this section, we will prove several relationships we used in Chapter 1. Please add these Theorems & Postulates to your lists.

THEOREM

2.3 Right Angles Congruence Theorem

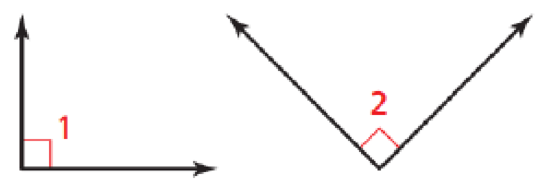
All right angles are congruent.

Proof: When proving theorems, it's best to rewrite them as conditional statements. Then, use the hypothesis as the "Given" and the conclusion as the "Prove/Show."

Theorem 2.3 - If two angles are right anges, then they are congruent.

Given $\angle 1$ and $\angle 2$ are right angles.

Prove $\angle 1 \cong \angle 2$



Two-Column Proof

STATEMENTS	REASONS
1. $\angle 1$ and $\angle 2$ are right angles.	**1.** Given
2. $m\angle 1 = 90°$, $m\angle 2 = 90°$	**2.** Definition of right angle
3. $m\angle 1 = m\angle 2$	**3.** Transitive Property of Equality
4. $\angle 1 \cong \angle 2$	**4.** Definition of congruent angles

Another proof format – <u>Flowchart proof</u> (compare to two-column form above)

$\angle 1$ and $\angle 2$ are right angles.

↓ Given

$m\angle 1 = 90°$, $m\angle 2 = 90°$ → $m\angle 1 = m\angle 2$ → $\angle 1 \cong \angle 2$

Definition of right angle | Transitive Property of Equality | Definition of congruent angles

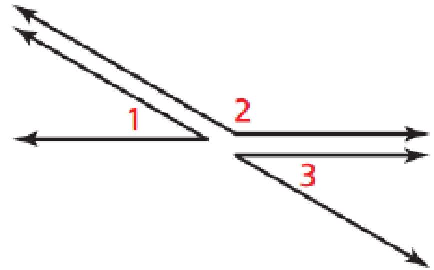
THEOREMS

2.4 Congruent Supplements Theorem

If two angles are supplementary to the same angle (or to congruent angles), then they are congruent.

If $\angle 1$ and $\angle 2$ are supplementary and $\angle 3$ and $\angle 2$ are supplementary, then $\angle 1 \cong \angle 3$.

Prove this Theorem Exercise 20 (case 2), page 179

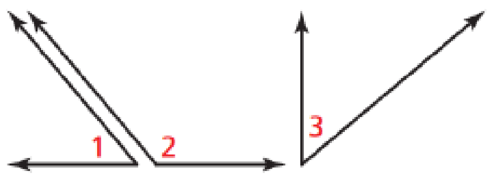


EXAMPLE 2 Proving a Case of the Congruent Supplements Theorem

Use the given two-column proof to write a flowchart proof that proves that two angles supplementary to the same angle are congruent.

Given $\angle 1$ and $\angle 2$ are supplementary.
$\angle 3$ and $\angle 2$ are supplementary.

Prove $\angle 1 \cong \angle 3$

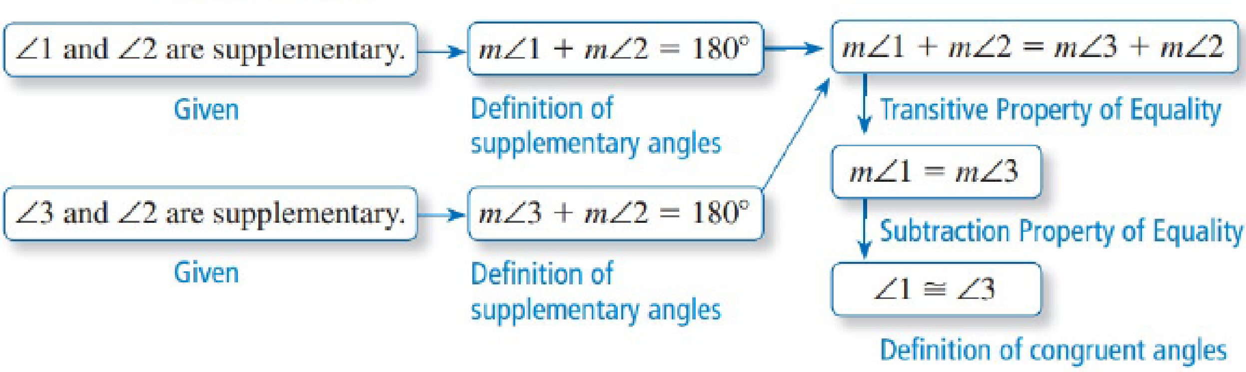


Two-Column Proof

STATEMENTS	REASONS
1. $\angle 1$ and $\angle 2$ are supplementary. $\angle 3$ and $\angle 2$ are supplementary.	**1.** Given
2. $m\angle 1 + m\angle 2 = 180^\circ$, $m\angle 3 + m\angle 2 = 180^\circ$	**2.** Definition of supplementary angles
3. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$	**3.** Transitive Property of Equality
4. $m\angle 1 = m\angle 3$	**4.** Subtraction Property of Equality
5. $\angle 1 \cong \angle 3$	**5.** Definition of congruent angles

Now in flowchart form:

Flowchart Proof

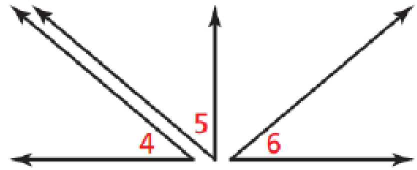


Another Theorem – This one will be proven with a third proof format – a Paragraph Proof

2.5 Congruent Complements Theorem

If two angles are complementary to the same angle (or to congruent angles), then they are congruent.

If ∠4 and ∠5 are complementary and ∠6 and ∠5 are complementary, then ∠4 ≅ ∠6.

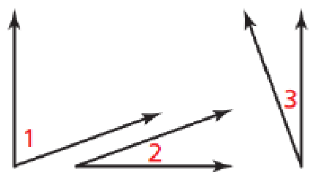


Prove this Theorem Exercise 19 (case 1), page 178; Exercise 24 (case 2), page 180

▶ **19. PROVING A THEOREM** Complete the paragraph proof for the Congruent Complements Theorem. Then write a two-column proof. *(See Example 3.)*

Given ∠1 and ∠2 are complementary.
∠1 and ∠3 are complementary.

Prove ∠2 ≅ ∠3



∠1 and ∠2 are complementary, and ∠1 and ∠3 are complementary. By the definition of ________________________ angles, $m\angle 1 + m\angle 2 = 90°$ and ________________________ = 90°. By the ______________________________, $m\angle 1 + m\angle 2 = m\angle 1 + m\angle 3$. By the Subtraction Property of Equality, ________________________. So, ∠2 ≅ ∠3 by the definition of ________________________.

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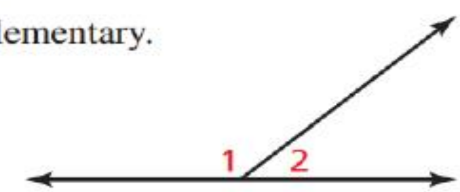
Please add the following old relationships to your Theorems & Postulates Lists:

POSTULATE AND THEOREM

2.8 Linear Pair Postulate

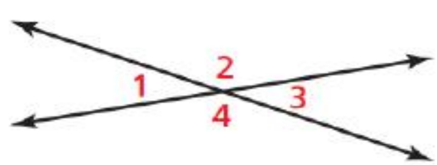
If two angles form a linear pair, then they are supplementary.

∠1 and ∠2 form a linear pair, so ∠1 and ∠2 are supplementary and $m\angle 1 + m\angle 2 = 180°$.



2.6 Vertical Angles Congruence Theorem

Vertical angles are congruent.

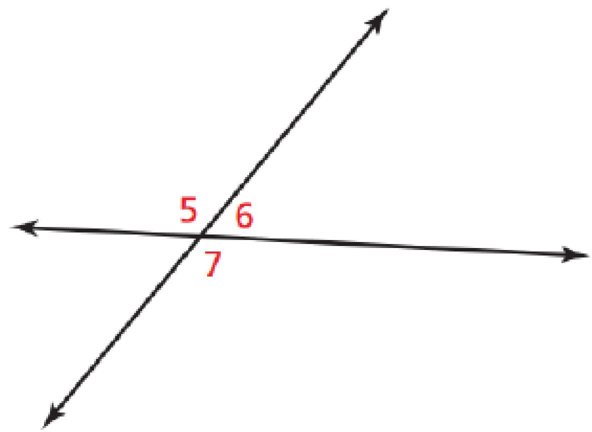


∠1 ≅ ∠3, ∠2 ≅ ∠4

EXAMPLE 3 Proving the Vertical Angles Congruence Theorem

WATCH

Use the given paragraph proof to write a two-column proof of the Vertical Angles Congruence Theorem.



Given $\angle 5$ and $\angle 7$ are vertical angles.

Prove $\angle 5 \cong \angle 7$

Paragraph Proof

$\angle 5$ and $\angle 7$ are vertical angles formed by intersecting lines. As shown in the diagram, $\angle 5$ and $\angle 6$ are a linear pair, and $\angle 6$ and $\angle 7$ are a linear pair. Then, by the Linear Pair Postulate, $\angle 5$ and $\angle 6$ are supplementary and $\angle 6$ and $\angle 7$ are supplementary. So, by the Congruent Supplements Theorem, $\angle 5 \cong \angle 7$.

Two-Column Proof

STATEMENTS	REASONS
1. $\angle 5$ and $\angle 7$ are vertical angles.	**1.** Given
2. $\angle 5$ and $\angle 6$ are a linear pair. $\angle 6$ and $\angle 7$ are a linear pair.	**2.** Definition of linear pair, as shown in the diagram
3. $\angle 5$ and $\angle 6$ are supplementary. $\angle 6$ and $\angle 7$ are supplementary.	**3.** Linear Pair Postulate
4. $\angle 5 \cong \angle 7$	**4.** Congruent Supplements Theorem